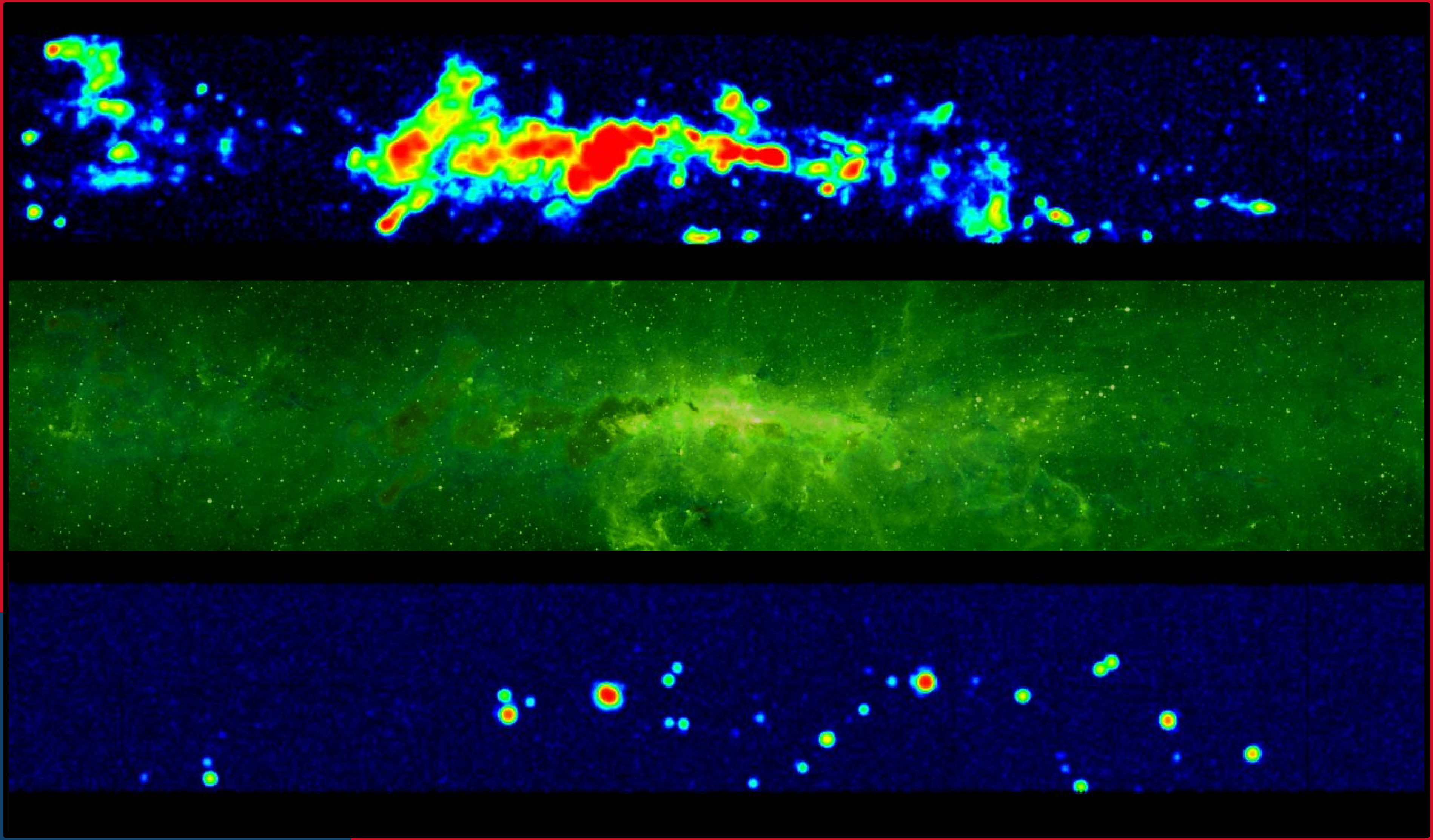
Studying Gas in our Galaxy at Different Wavelengths



The Sydney Institute for Astrophysics

Cormac Purcell, Harley Wood Winter School, 2012-June-27

Talk Outline

- Why study molecular gas in the Galaxy?
- A little bit of theory
  - Spectra from rotating molecules
  - Radiation transport in the ISM
  - Getting physical parameters from observations
- Putting it all together – real world examples

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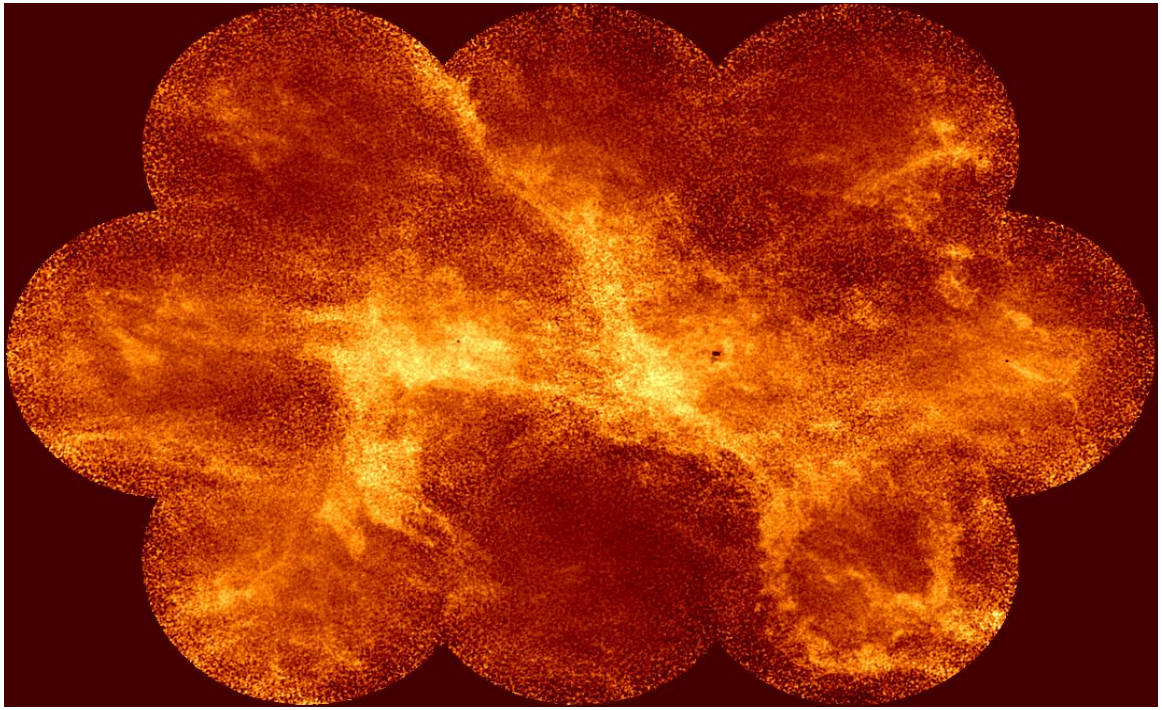


Image: DRAO

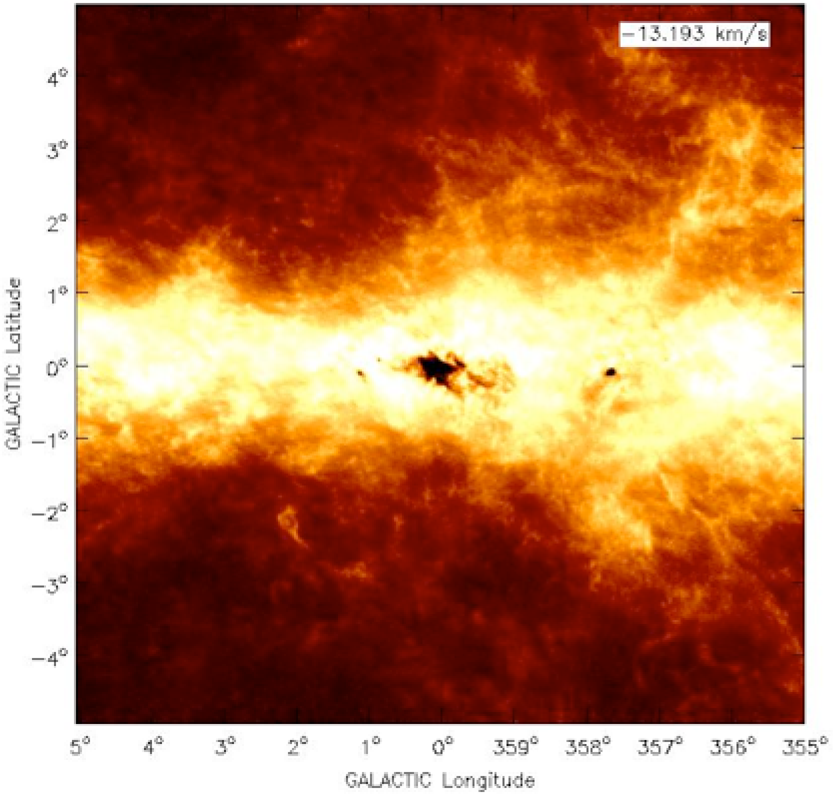
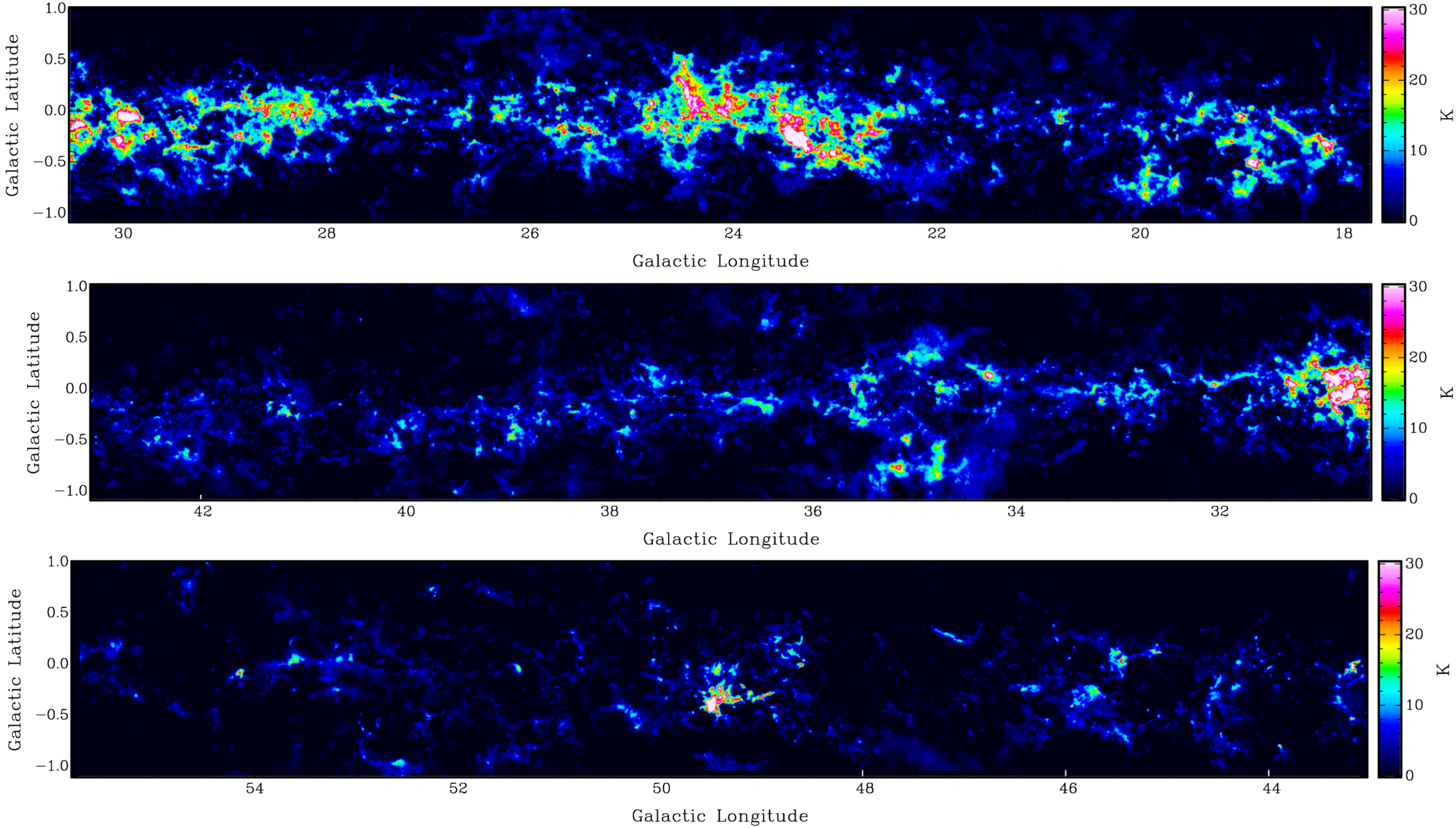


Image: McClure-Griffiths

Why study molecular gas in the Galaxy?

- CO a great proxy for H_2: BU-FCRAO Galactic Ring Survey (Jackson et al 2006)



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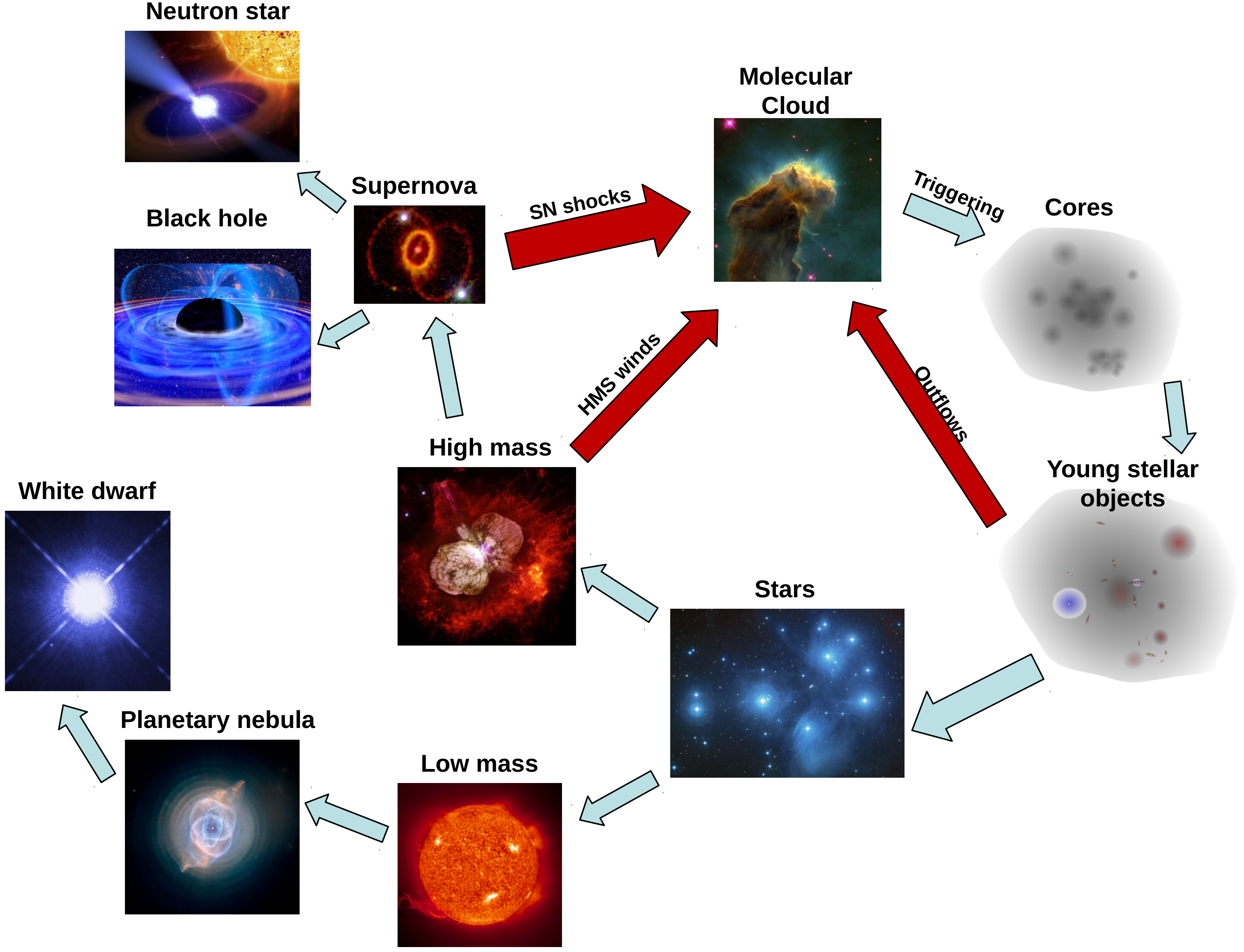

Figure: Andrew Walsh

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Why study molecular gas in the Galaxy?

Molecules in the Interstellar Medium or Circumstellar Shells (as of 05/2012)

2 atoms	3 atoms	4 atoms	5 atoms	6 atoms	7 atoms	8 atoms	9 atoms	10 atoms	11 atoms	12 atoms	>12 atoms
H_2	C_3 *	*c*-C_3H	C_5 *	C_5H	C_6H	CH_3C_3N	CH_3C_4H	CH_3C_5N	HC_9N	*c*-C_6H_6 *	$HC_{11}N$
AlF	C_2H	*l*-C_3H	C_4H	*l*-H_2C_4	CH_2CHCN	$HC(O)OCH_3$	CH_3CH_2CN	$(CH_3)_2CO$	CH_3C_6H	$C_2H_5OCH_3$?	C_{60} * 2012
AlCl	C_2O	C_3N	C_4Si	C_2H_4*	CH_3C_2H	CH_3COOH	$(CH_3)_2O$	$(CH_2OH)_2$	C_2H_5OCHO	*n*-C_3H_7CN	C_{70} *
C_2**	C_2S	C_3O	*l*-C_3H_2	CH_3CN	HC_5N	C_7H	CH_3CH_2OH	CH_3CH_2CHO			
CH	CH_2	C_3S	*c*-C_3H_2	CH_3NC	CH_3CHO	H_2C_6	HC_7N				
CH^+	HCN	C_2H_2*	H_2CCN	CH_3OH	CH_3NH_2	CH_2OHCHO	C_8H				
CN	HCO	NH_3	CH_4 *	CH_3SH	*c*-C_2H_4O	*l*-HC_6H *	$CH_3C(O)NH_2$				
CO	HCO^+	HCCN	HC_3N	HC_3NH^+	H_2CCHOH	CH_2CHCHO (?)	C_8H^-				
CO^+	HCS^+	$HCNH^+$	HC_2NC	HC_2CHO	C_6H^-	CH_2CCHCN	C_3H_6				
CP	HOC^+	HNCO	HCOOH	NH_2CHO		H_2NCH_2CN					
SiC	H_2O	HNCS	H_2CNH	C_5N							
HCl	H_2S	$HOCO^+$	H_2C_2O	*l*-HC_4H *							
KCl	HNC	H_2CO	H_2NCN	*l*-HC_4N							
NH	HNO	H_2CN	HNC_3	*c*-H_2C_3O							
NO	MgCN	H_2CS	SiH_4 *	H_2CCNH (?)							
NS	MgNC	H_3O^+	H_2COH^+	C_5N^-							
NaCl	N_2H^+	*c*-SiC_3	C_4H^-								
OH	N_2O	CH_3 *	HC(O)CN								
PN	NaCN	C_3N^-									
SO	OCS	PH_3 ?									
SO^+	SO_2	HCNO									
SiN	*c*-SiC_2	HOCN									

http://www.astro.uni-koeln.de/cdms/molecules

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SiO	CO_2 *	HSCN									
SiS	NH_2	H_2O_2 2011									
CS	H_3^+ *										
HF	H_2D^+, HD_2^+										
HD	SiCN										
FeO ?	AlNC										
O_2	SiNC										
CF^+	HCP										
SiH ?	CCP										
PO	AlOH										
AlO	H_2O^+										
OH^+	H_2Cl^+										
CN^-	KCN										
SH^+ 2011	FeCN 2011										
SH 2012	HO_2 2012										
HCl^+ 2012											

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CF^+	HCP										
SiH ?	CCP										
PO	AlOH										
AlO	H_2O^+										
OH^+	H_2Cl^+										
CN^-	KCN										
SH^+ 2011	FeCN 2011										
SH 2012	HO_2 2012										
HCl^+ 2012											

Extragalactic Molecules (as of 05/2012)

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OH	H_2O	H_2CO	c-C_3H_2	CH_3OH	CH_3CCH	HC_6H	c-C_6H_6 *
CO	HCN	NH_3	HC_3N	CH_3CN	CH_3NH_3 2011		C_{60}* *? 2012
H_2 *	HCO^+	HNCO	CH_2NH	HC_4H *	CH_3CHO 2011		
CH **	C_2H	C_2H_2 *	NH_2CN				
CS	HNC	H_2CS ?	l-C_3H_2 2011				
CH^+ **	N_2H^+	$HOCO^+$	H_2CCN 2011				
CN	OCS	c-C_3H	H_2CCO 2011				
SO	HCO	H_3O^+	C_4H 2011				
SiO	H_2S	l-C_3H 2011					
CO^+	SO_2						
NO	HOC^+						
NS	C_2S						
NH	H_2O^+						
OH^+							
HF							
SO^+ 2011							

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Why study molecular gas in the Galaxy?

Image: NASA

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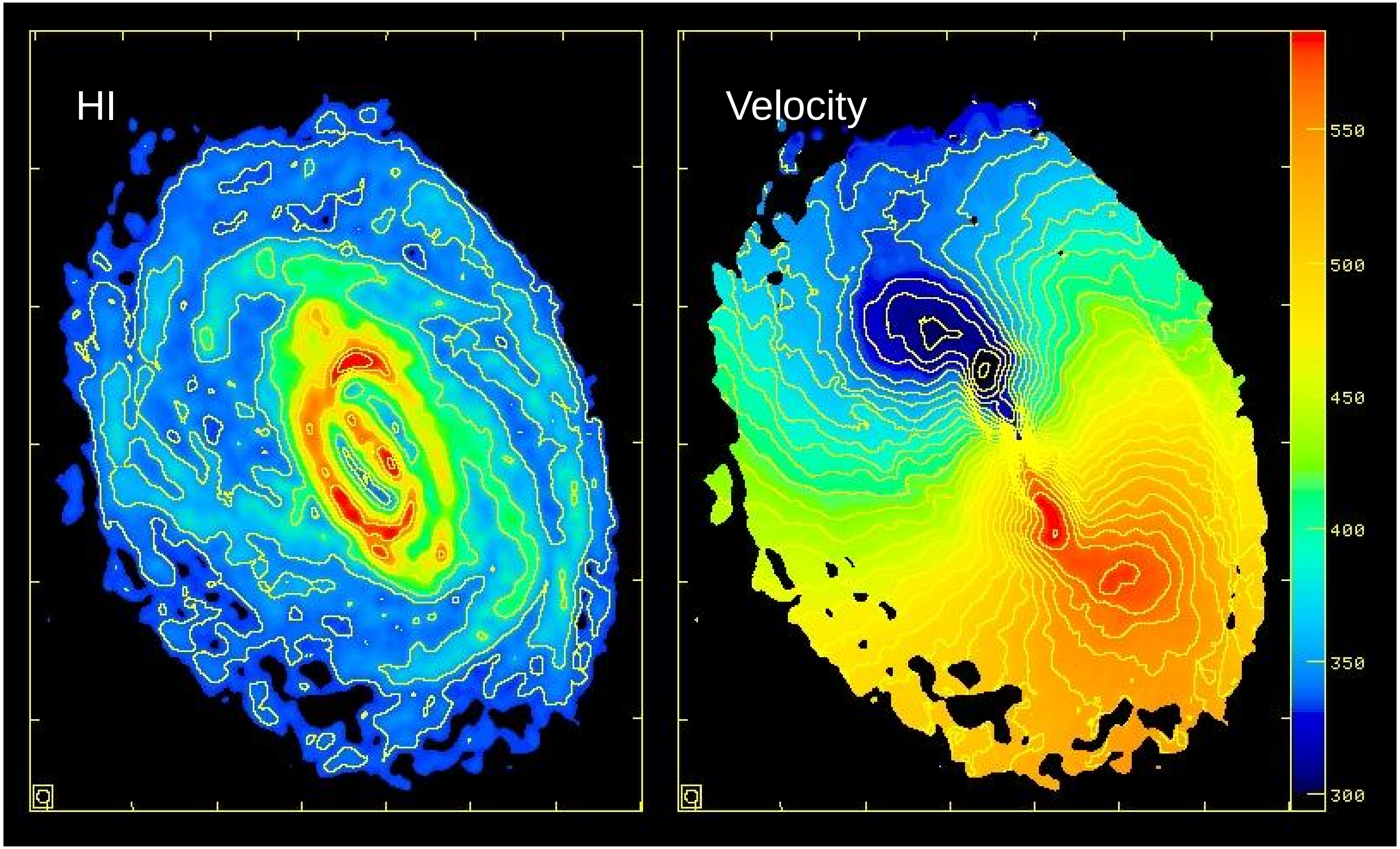

Image: B. Koribalski

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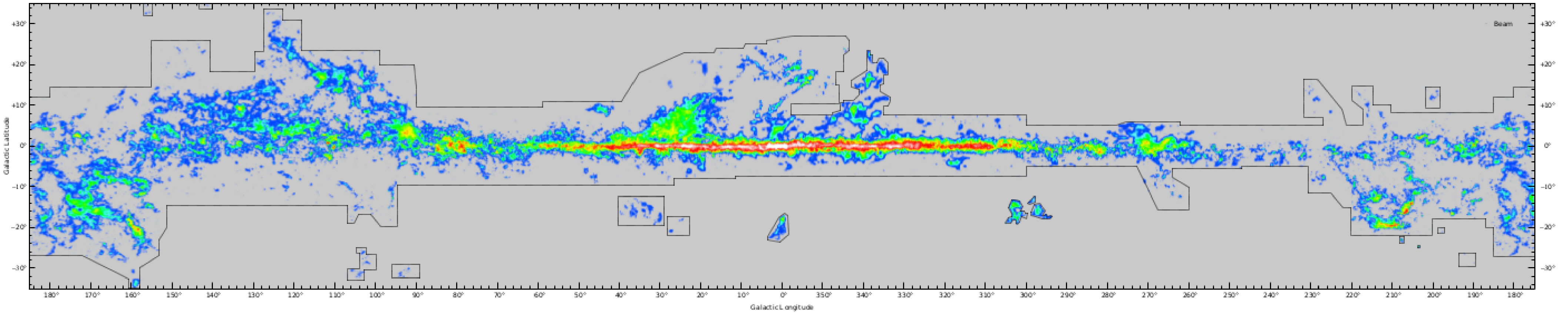

Dame et al 2001

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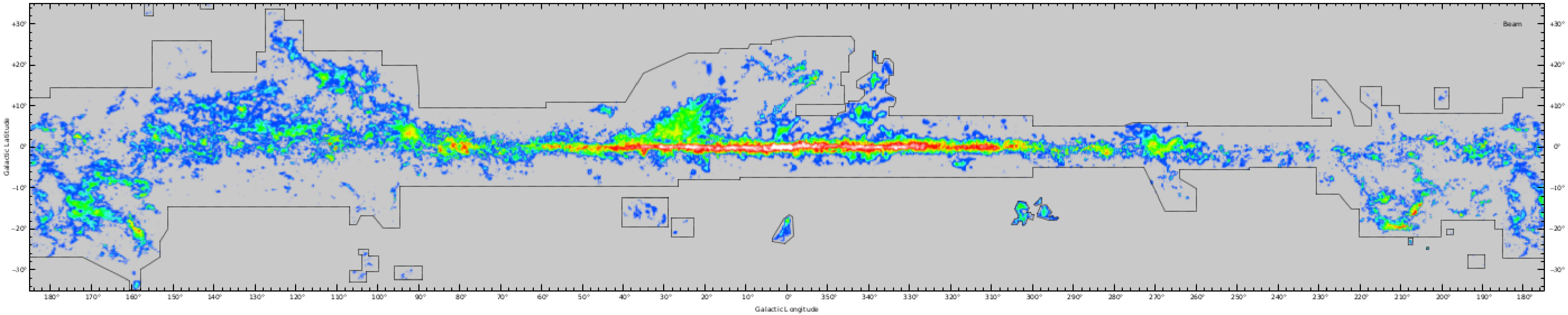

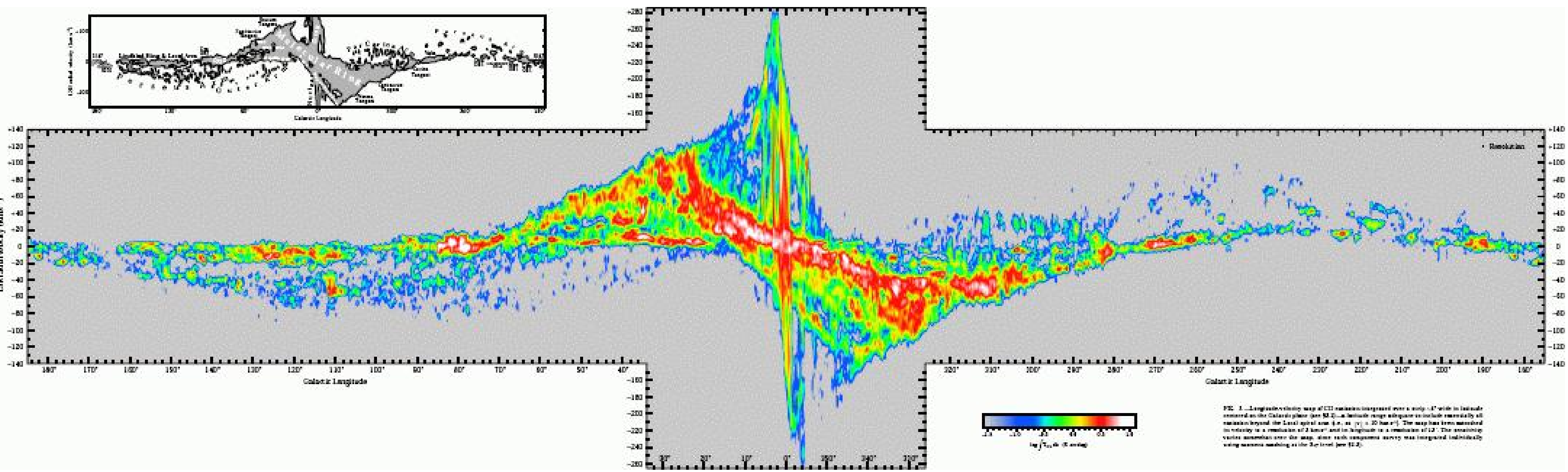

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- Molecular gas is beautiful!

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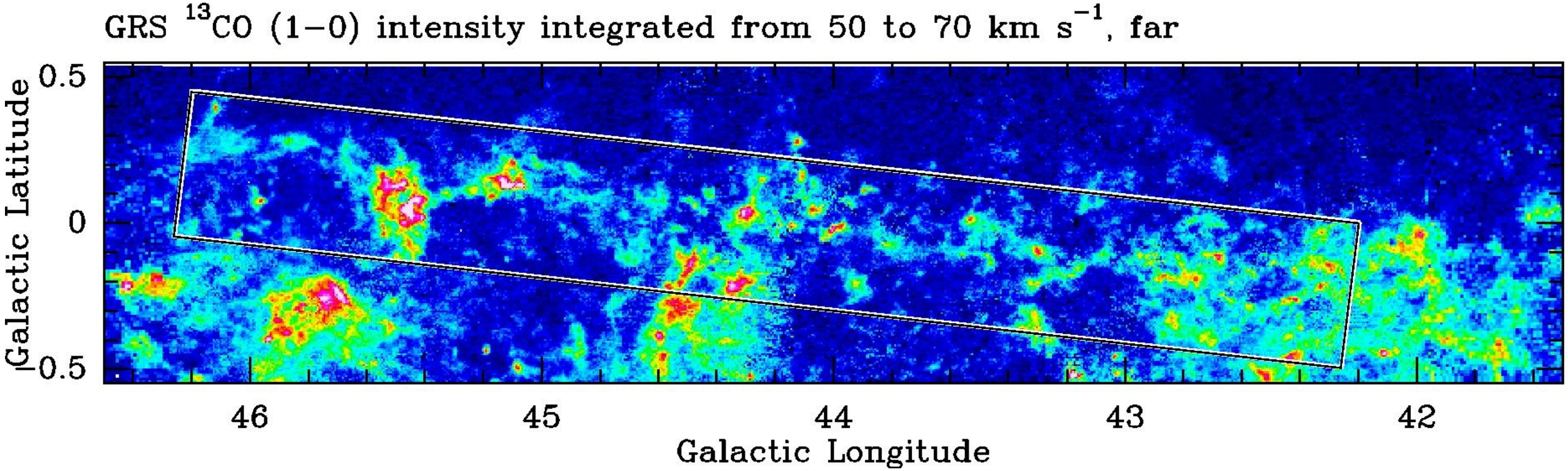

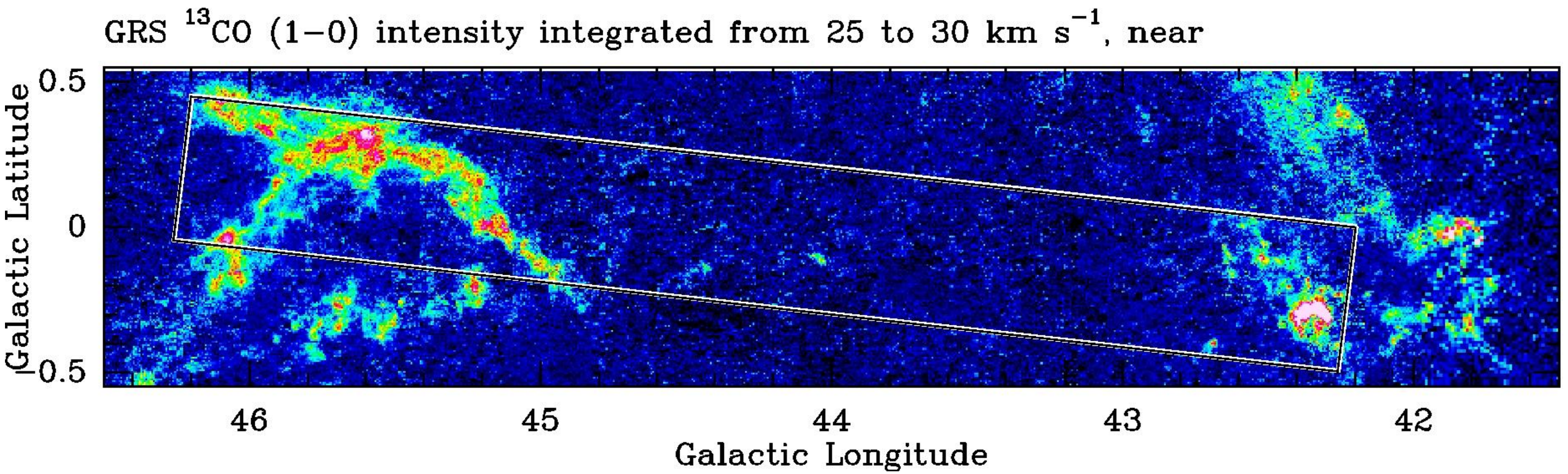

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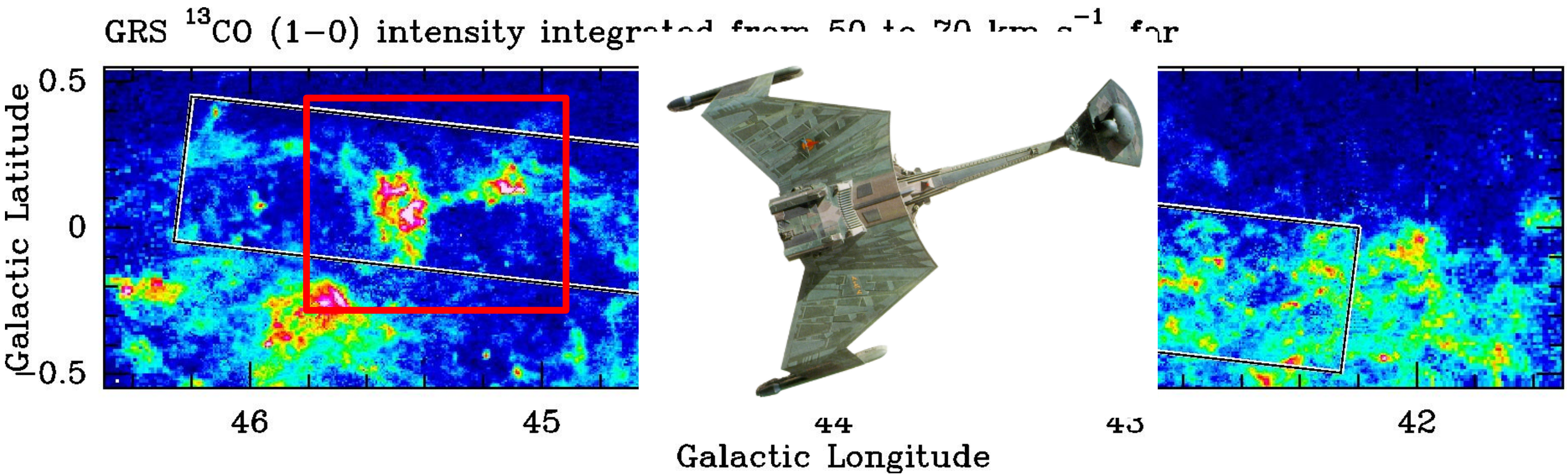

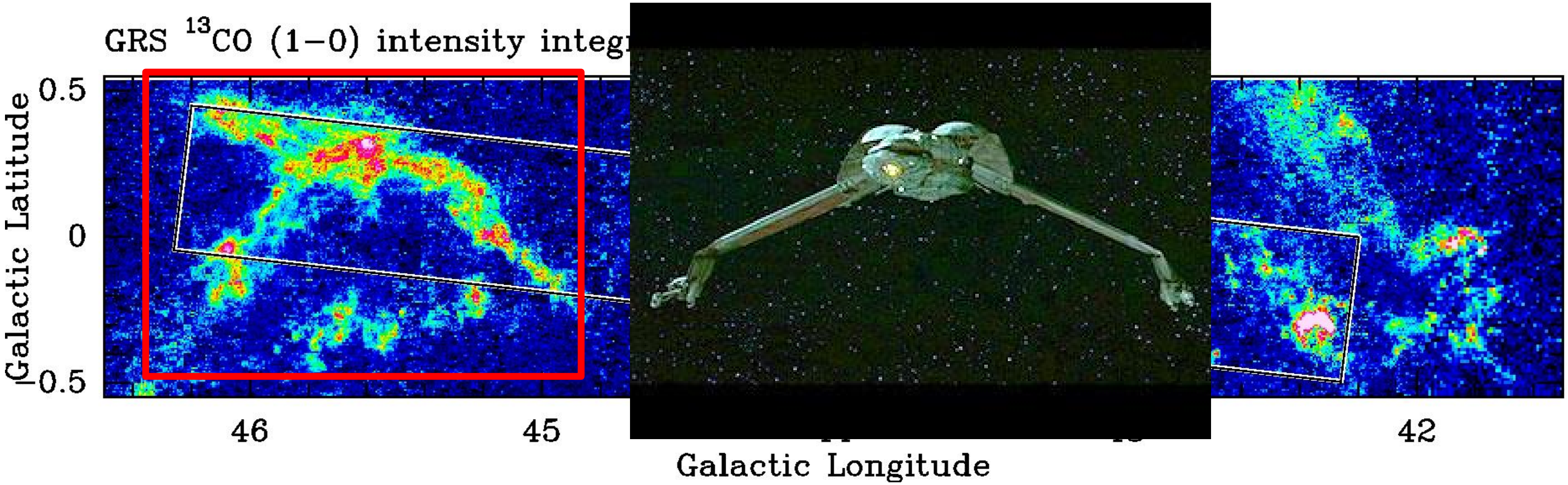

A little bit of Theory

Emission from molecules

- Molecular transitions fall into three energy bins:
  - Electronic: $\Delta E \approx$ a few eV, visible or UV emission lines
  - Vibrational (nuclear vibrations): $\Delta E \approx 10^{-1}$ to 10^{-2} eV, infrared lines
  - Rotational: $\Delta E \approx 10^{-1}$ eV, radio emission lines (cm to mm wavelengths)

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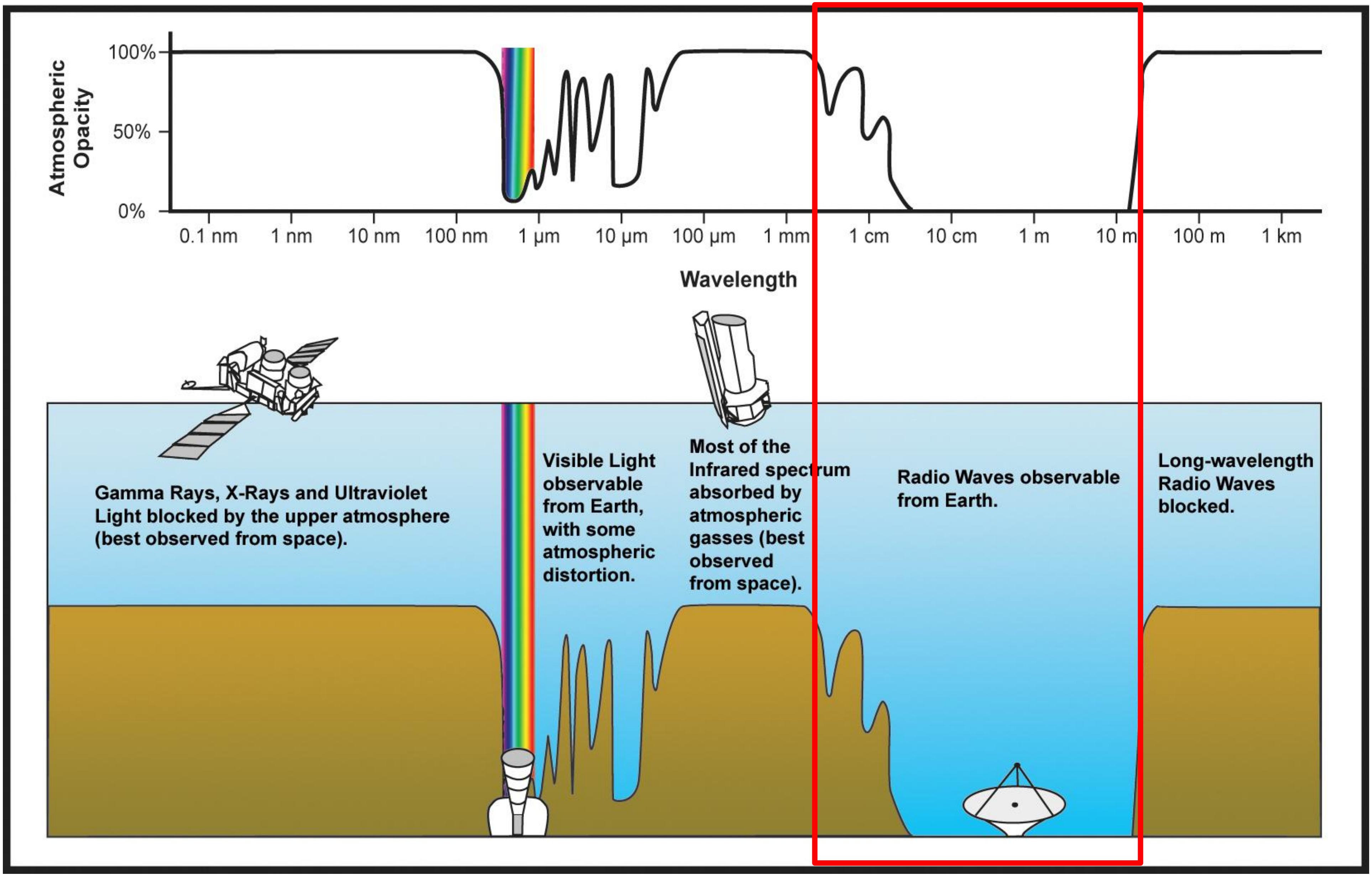

Image: NASA/IPAC

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Emission from **rotating** molecules

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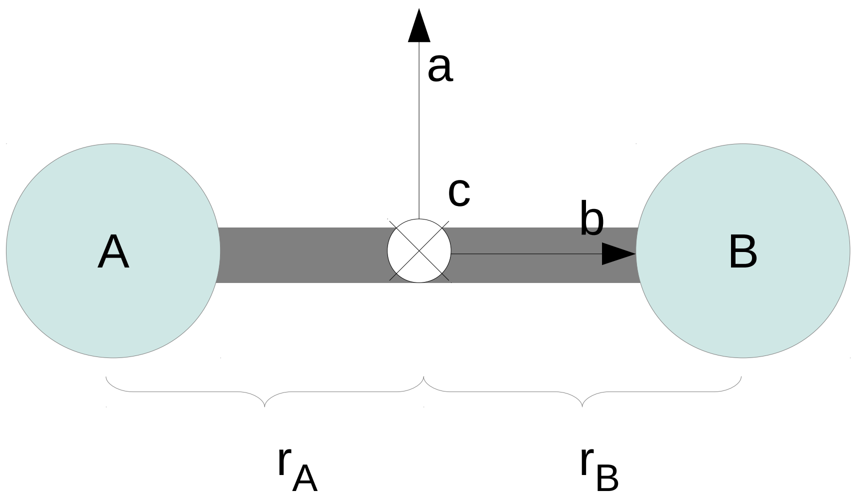
- Classical picture of rotation:
  - Moment of inertia I around axis i

$$I = \sum m_i r_i^2,$$



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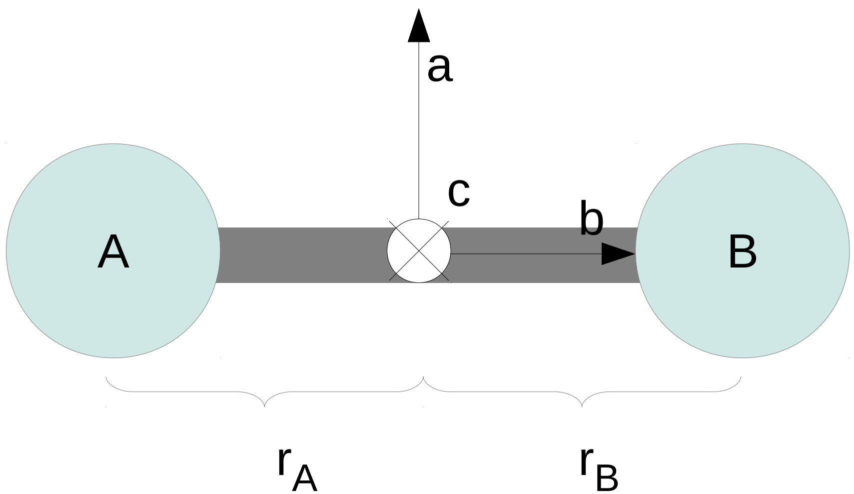
$$E = \frac{1}{2}\left[I_a \omega_a^2 + I_b \omega_b^2 + I_c \omega_c^2\right],$$



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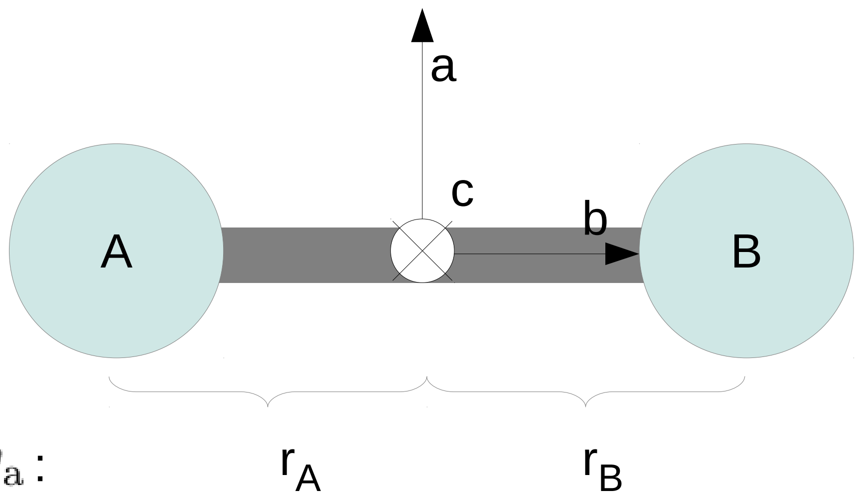
  - In terms of angular mometum $P_a = I_a \omega_a$:

$$E = \frac{P_a^2}{2I_a} + \frac{P_b^2}{2I_b} + \frac{P_c^2}{2I_c},$$



$$P^2 = P_a^2 + P_b^2 + P_c^2$$

A little bit of Theory

Emission from **rotating** molecules

- Four types of rotor configuration, grouped by symmetry:

Spherical Rotors:	$I_a = I_b = I_c$,	e.g., CH_4, SiH_4.
Linear Rotors:	$I_a = 0, I_b = I_c$,	e.g., CO, HCO+, HCN, HNC, N_2H^+
Symmetric Rotors:	$I_a = I_b \neq I_c$,	e.g., NH_3, CH_3CN, CH_3Cl.
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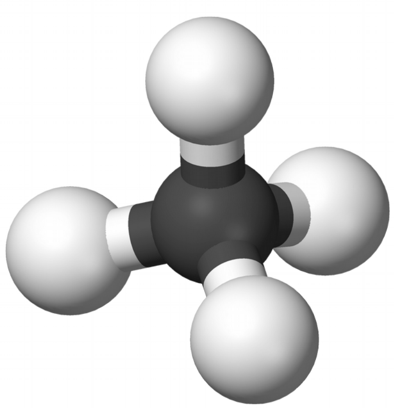

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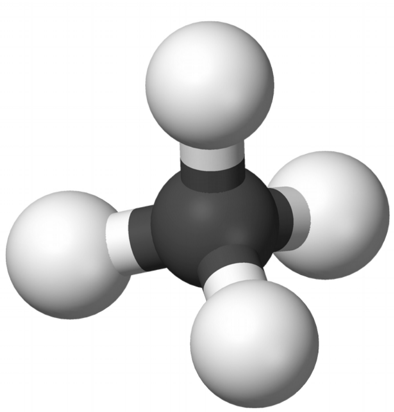

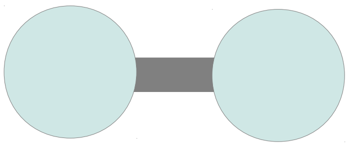

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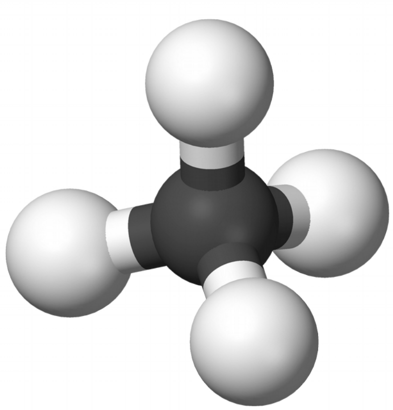
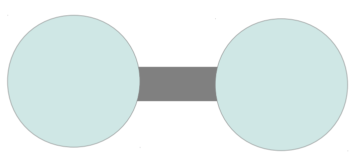
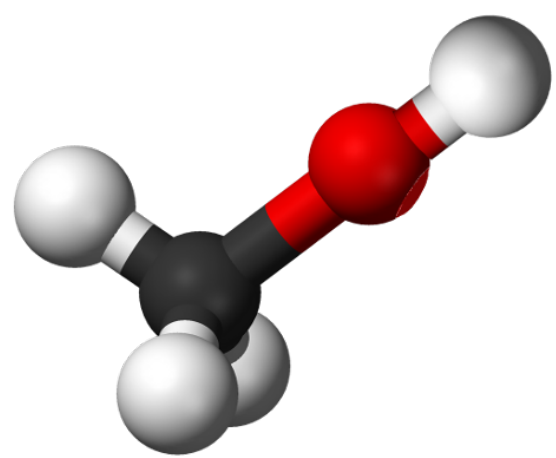

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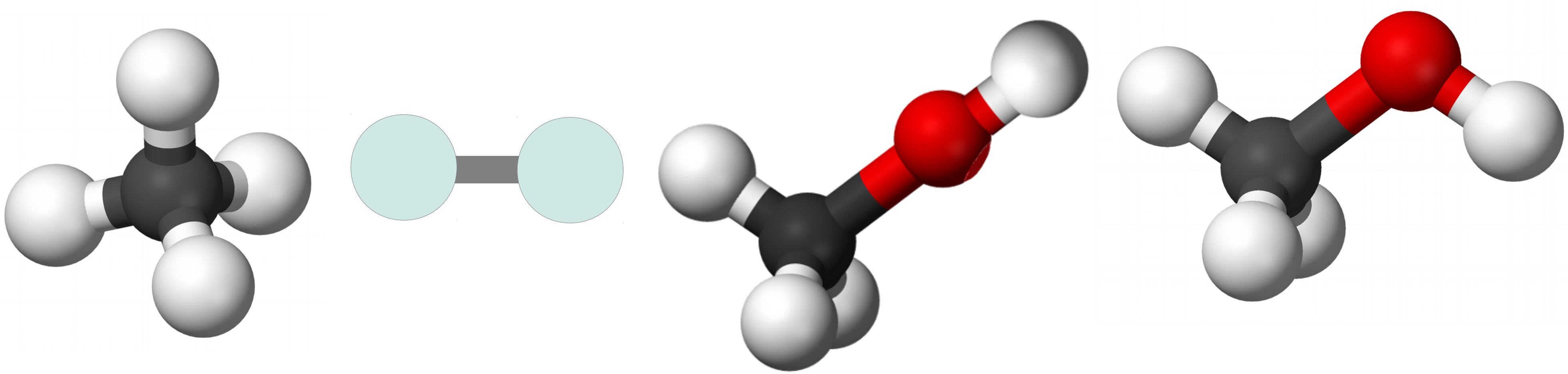

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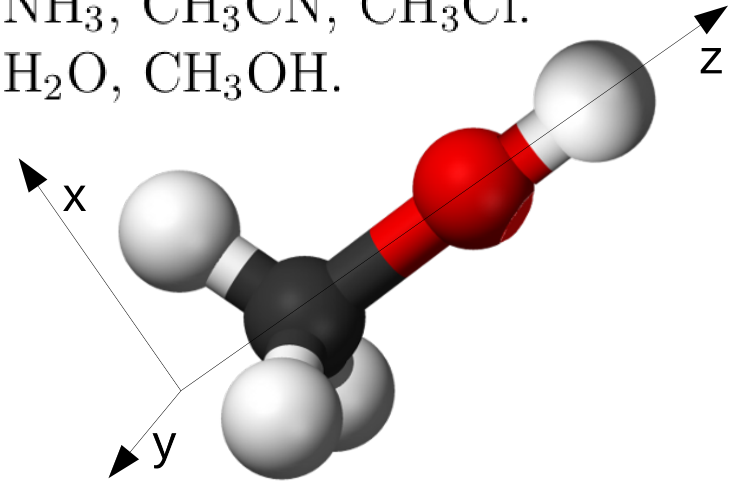
- For simplicity consider only symmetric rotors:

  one unique and two identical axes



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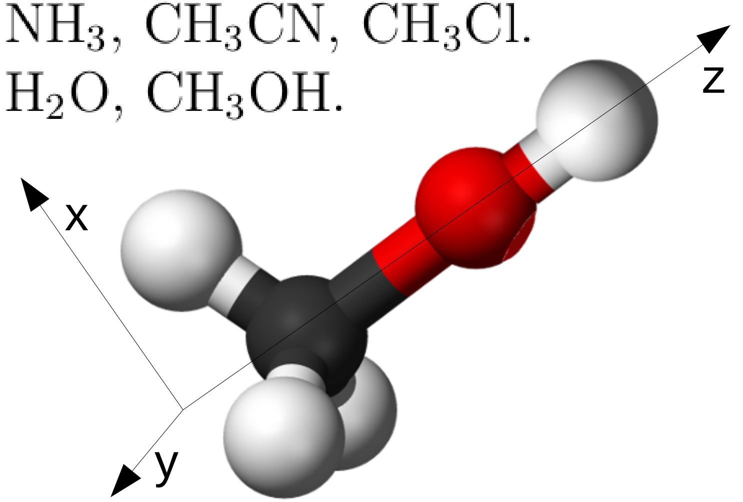

- To emit radiation the molecule must have a permanent dipole moment *μ*, arising from the asymmetric distribution of +ve and –ve charges on the molecule.

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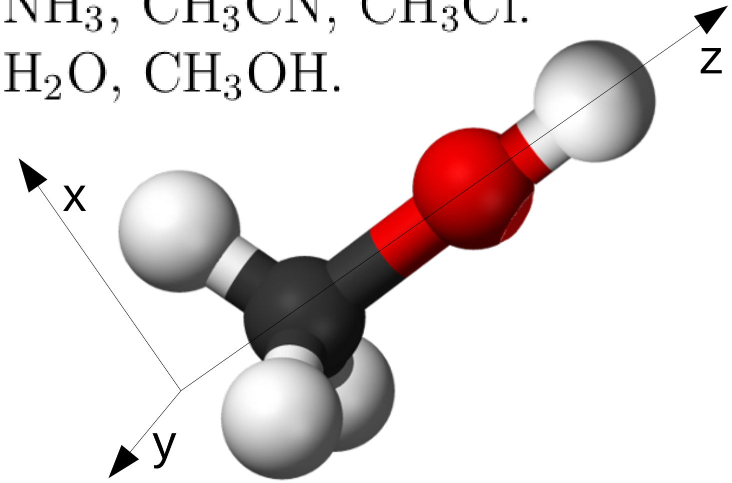

- To emit radiation the molecule must have a permanent dipole moment *μ*, arising from the asymmetric distribution of +ve and –ve charges on the molecule.

- H_2, the most abundant molecule, has a low *μ* and so can not usually emit

- CO is used as proxy assuming a constant ratio $[CO/H_2] = 10^{-4}$

A little bit of Theory

Emission from **rotating** molecules

- Energy levels in a classical **rigid** symmetric rotor given by:

$$E = \frac{P^2}{2I_\perp} - \frac{P_c^2}{2I_\perp} + \frac{P_c^2}{2I_\parallel} = \frac{P^2}{2I_\perp} + \left(\frac{1}{2I_\parallel} - \frac{1}{2I_\perp}\right) P_c^2$$

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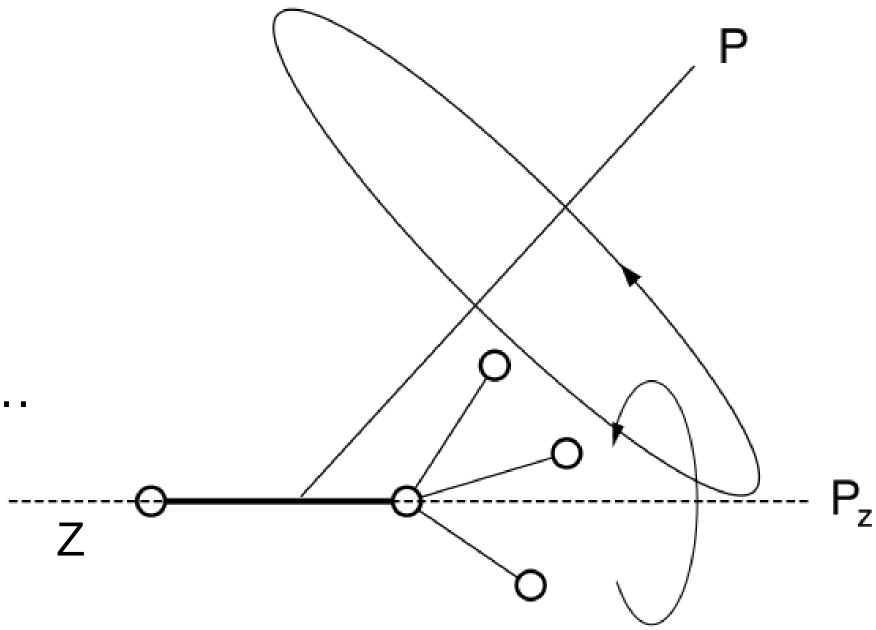
- The projection of the total angular momentum on to the principal axis is restricted to values of Kħ, with K=±0, ±1, …



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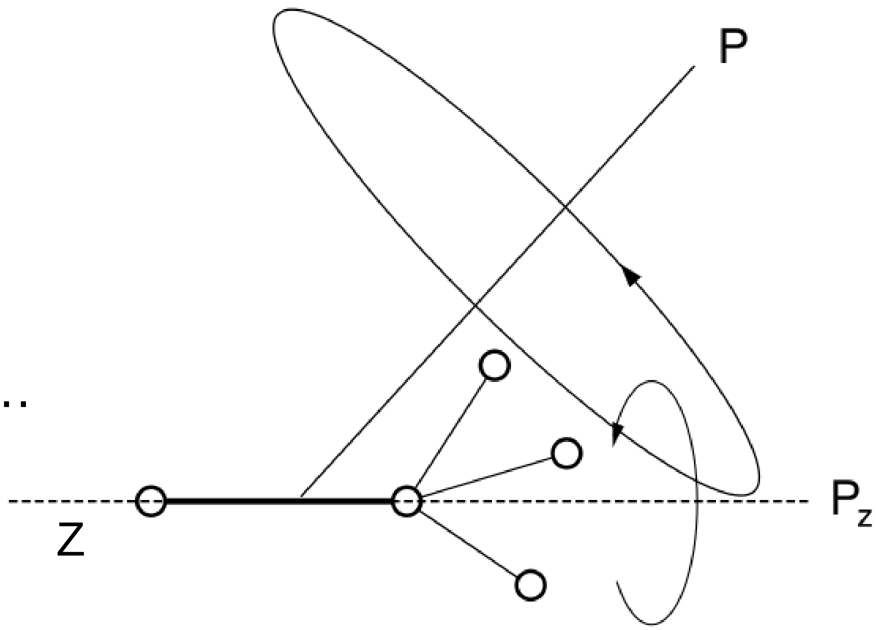
$$E_{J,K} = hBJ(J+1) + h(A-B)K^2$$



with $A = \frac{\hbar}{4\pi I_\parallel}$ and $B = \frac{\hbar}{4\pi I_\perp}$ being the rotational constants of the molecule

A little bit of Theory

Emission from **rotating** molecules

- For a simple rigid rotor the frequencies in a ΔJ +/- 1 transition are given by:

$\nu = 2\mathrm{B}(\mathrm{J} + 1)$ … at least to first order, as A-B is small.

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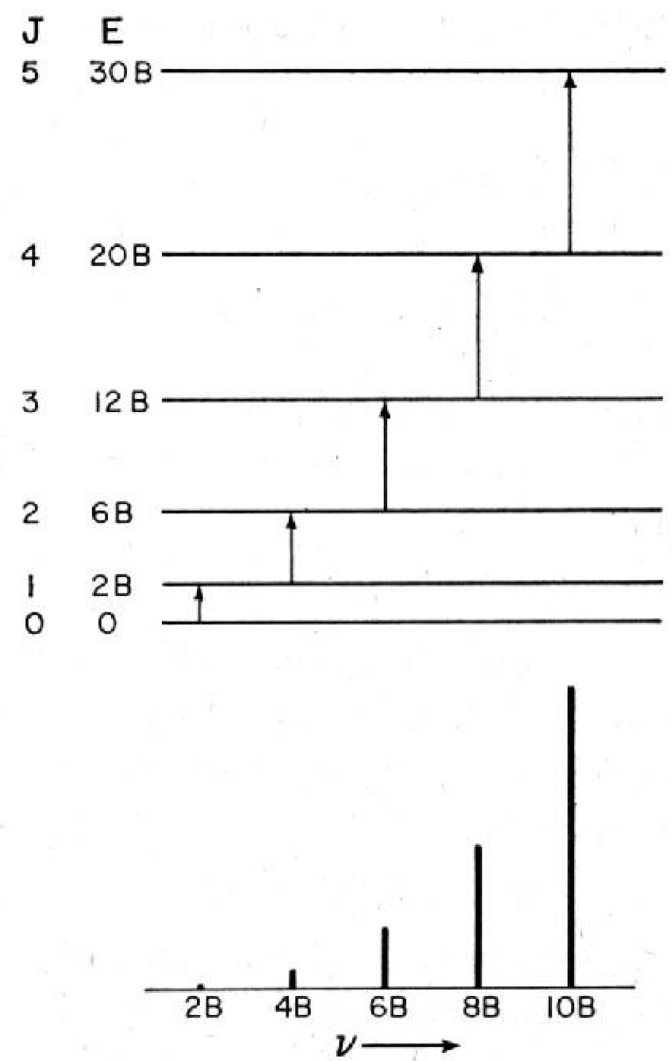

From Gordy & Cook 1970

A little bit of Theory

Emission from **rotating** molecules

- Molecules are not rigid rotors, but are affected by centripetal distortion
- Bonds lengthen, leading to a change in I and B.

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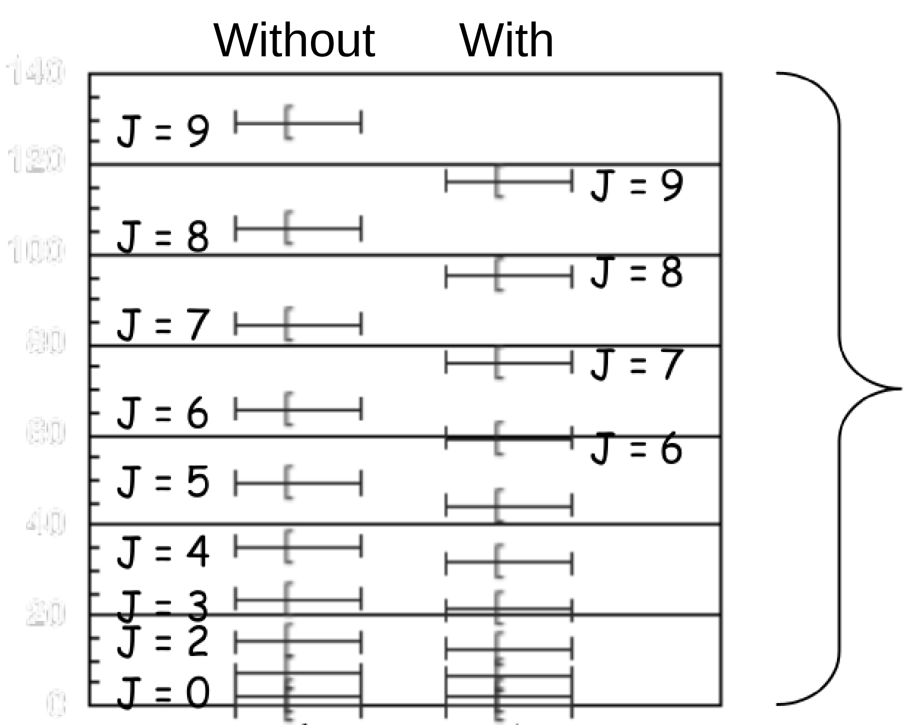
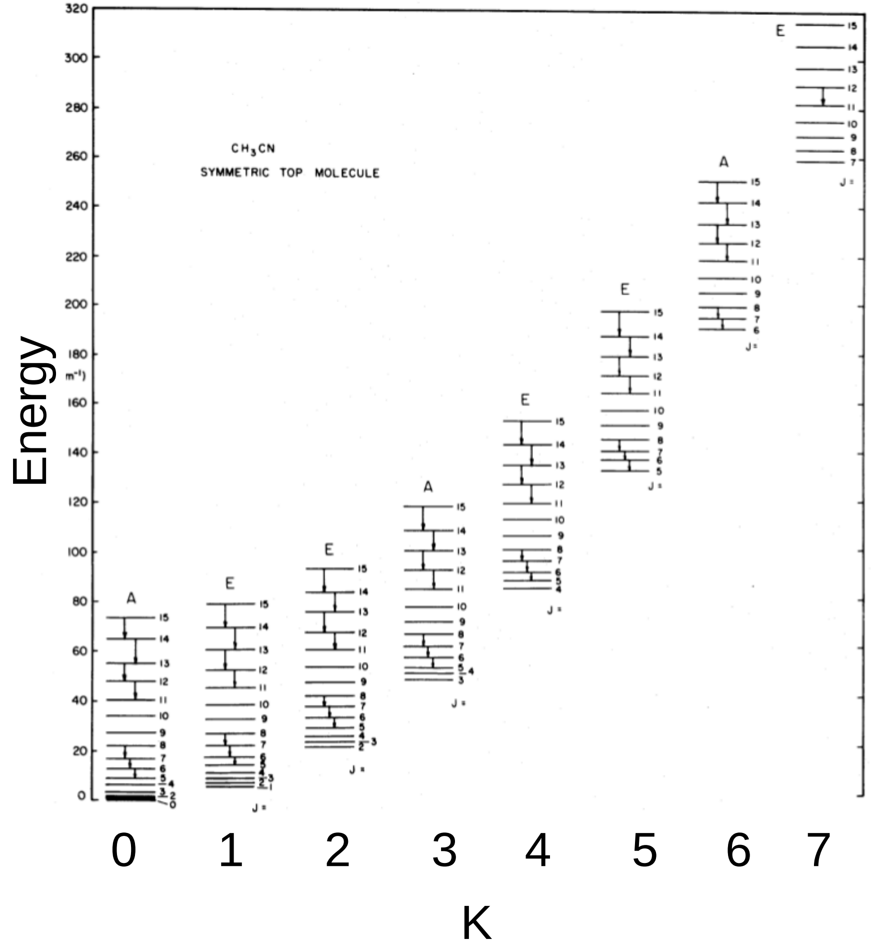


Greatly exaggerated stretching
- D/B = 0.1
- most molecules have D/B ~ 10^{-5}

A little bit of Theory

Emission from **rotating** molecules



A little bit of Theory

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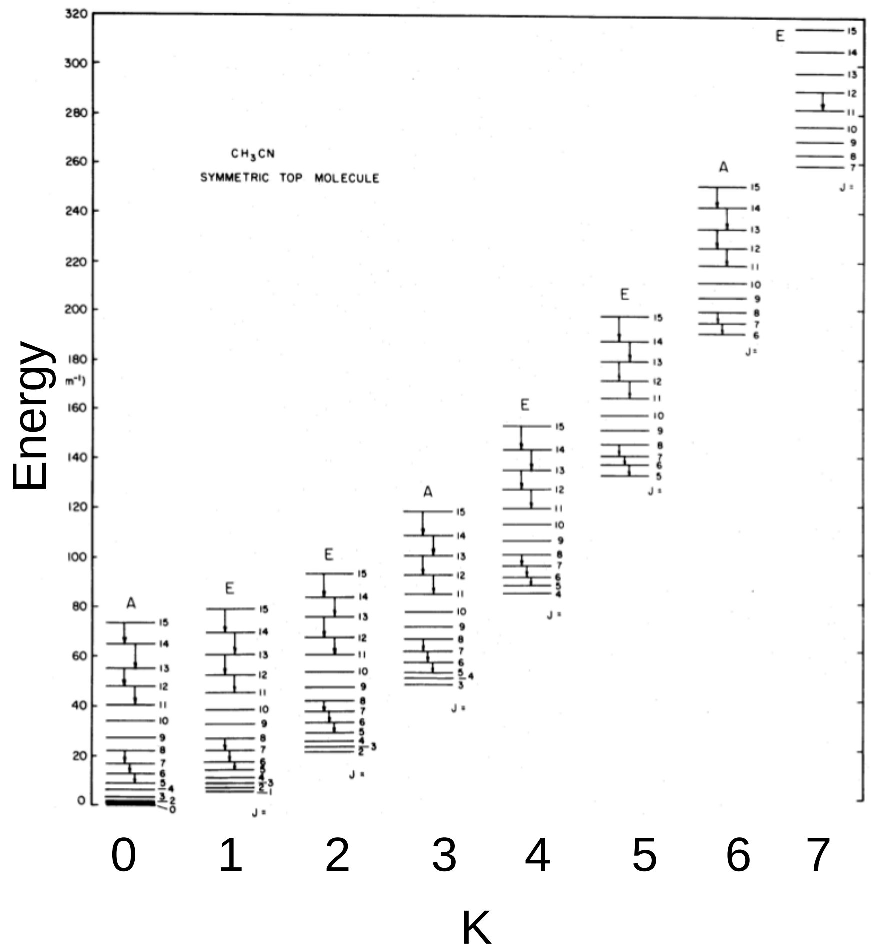
- Degenerate (overlapping) energy levels
- Energy of any J-level is degenerate by a factor $g_u = 2J + 1$



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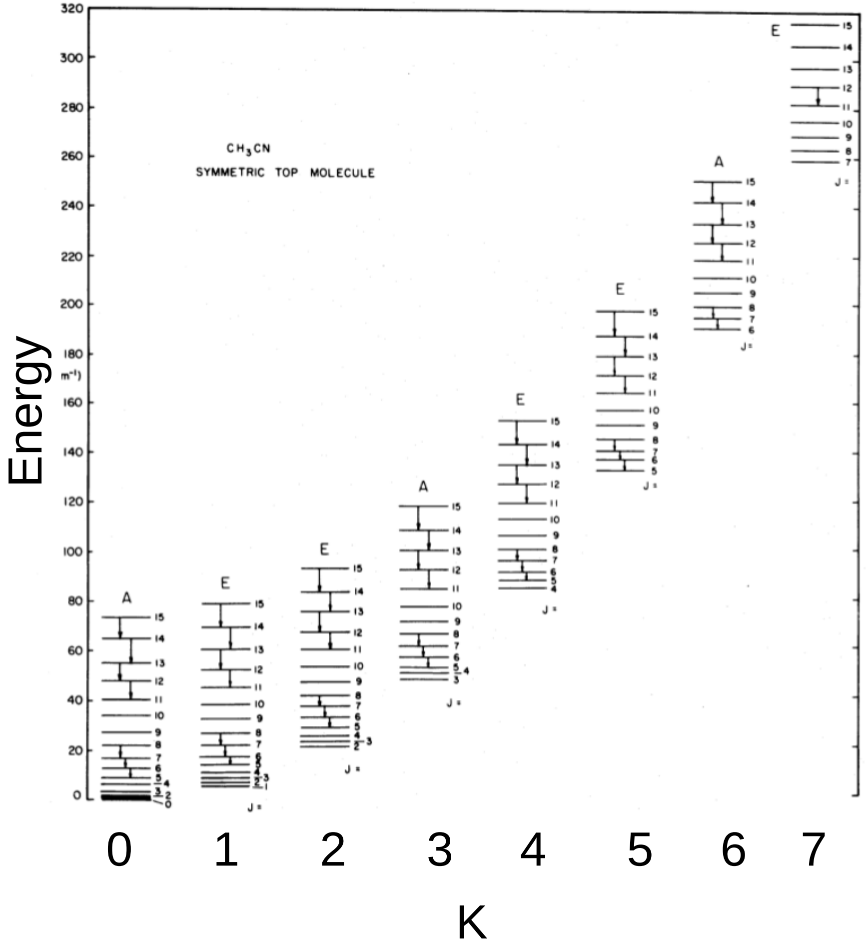

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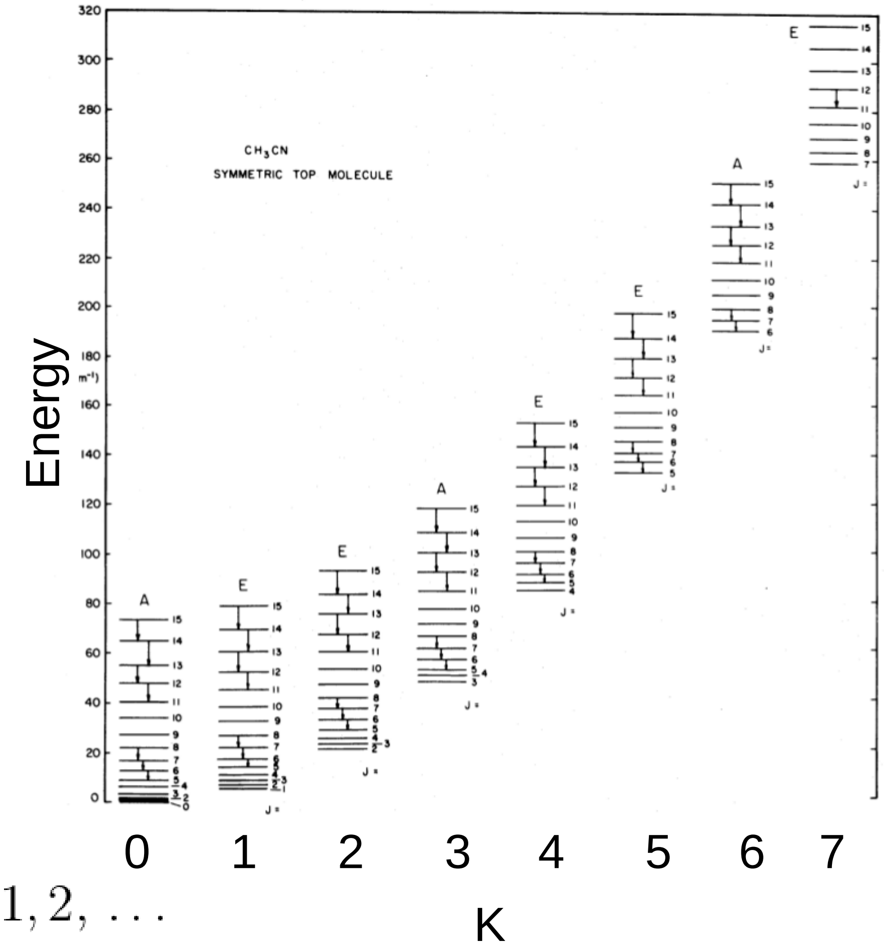

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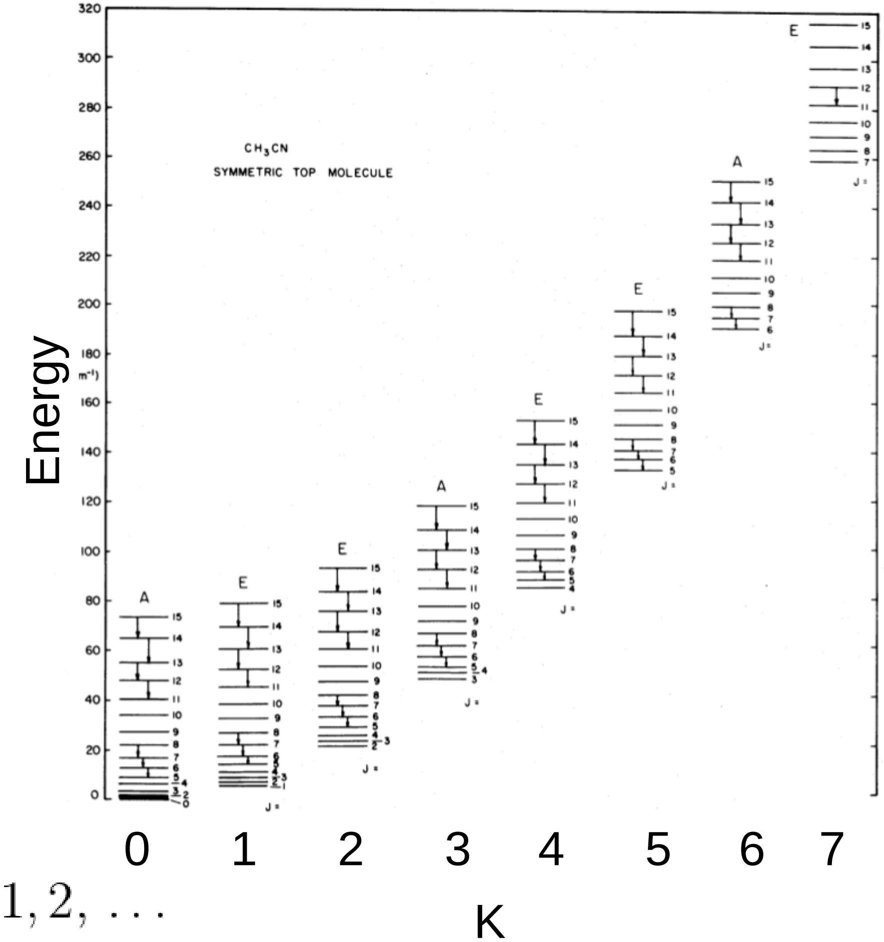

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- Total degeneracy of any J,K level is then: $g_u S(I, K)$

A little bit of Theory

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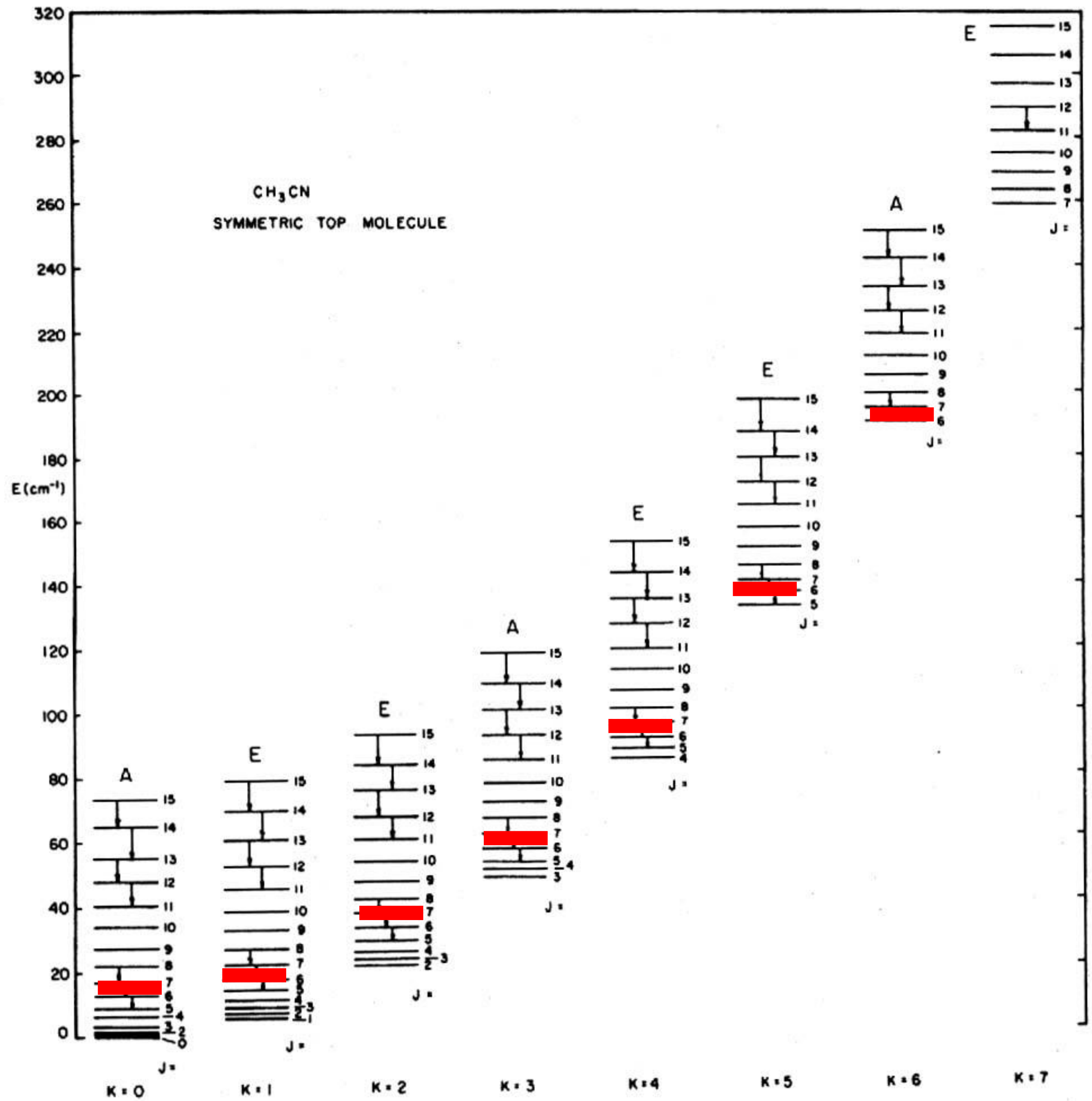

From Loren & Mundy 1984

A little bit of Theory

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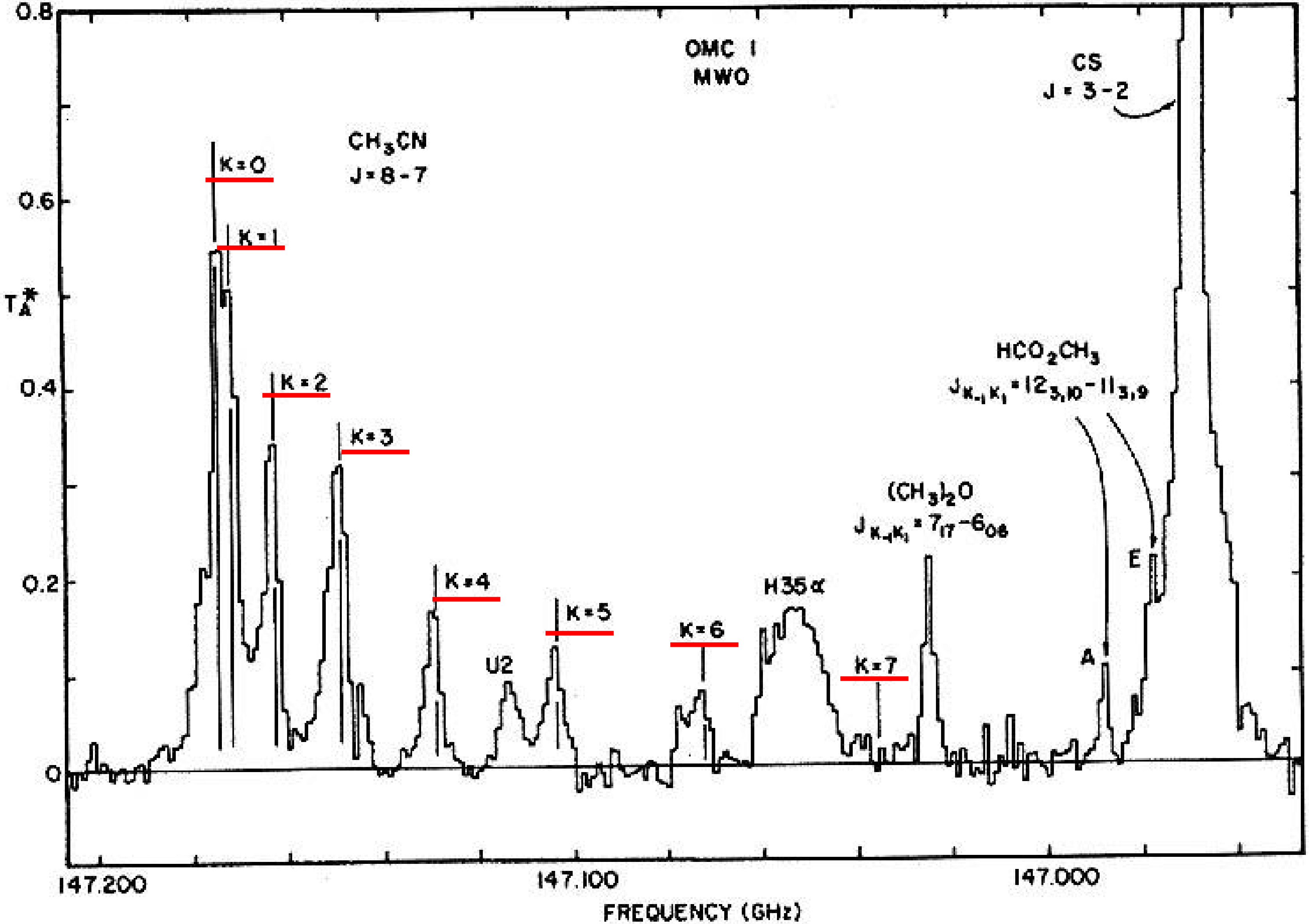

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- In a population the number of molecules at a particular rotational energy will be governed by the Boltzmann Distribution:

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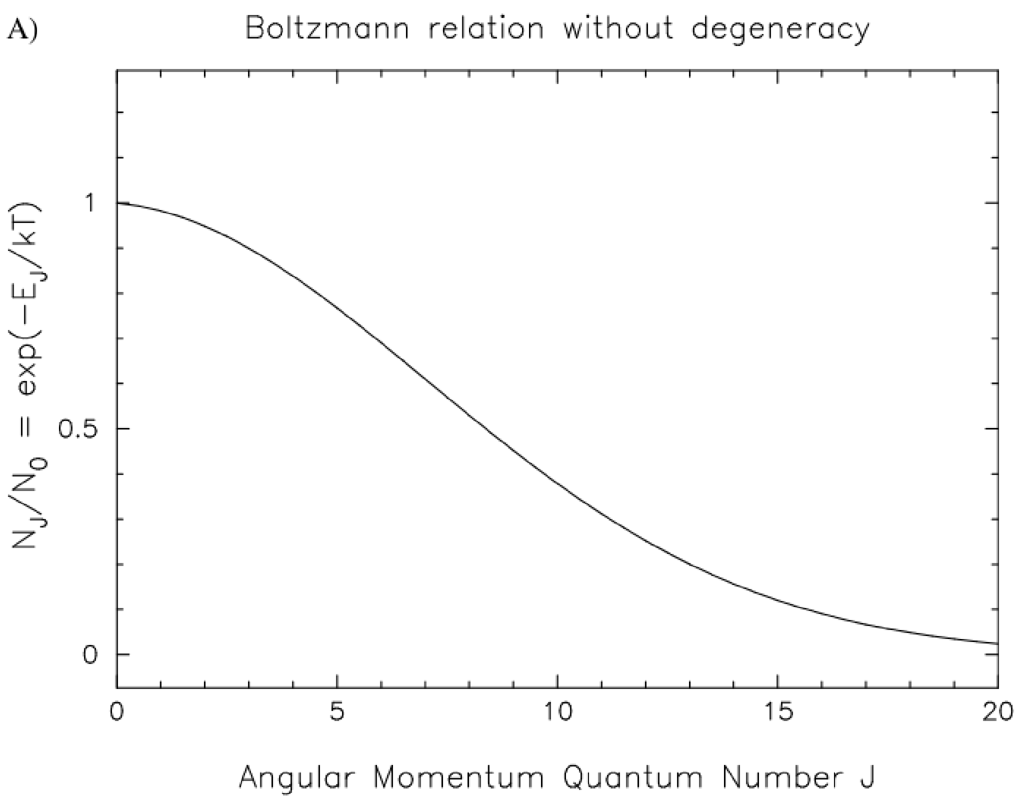

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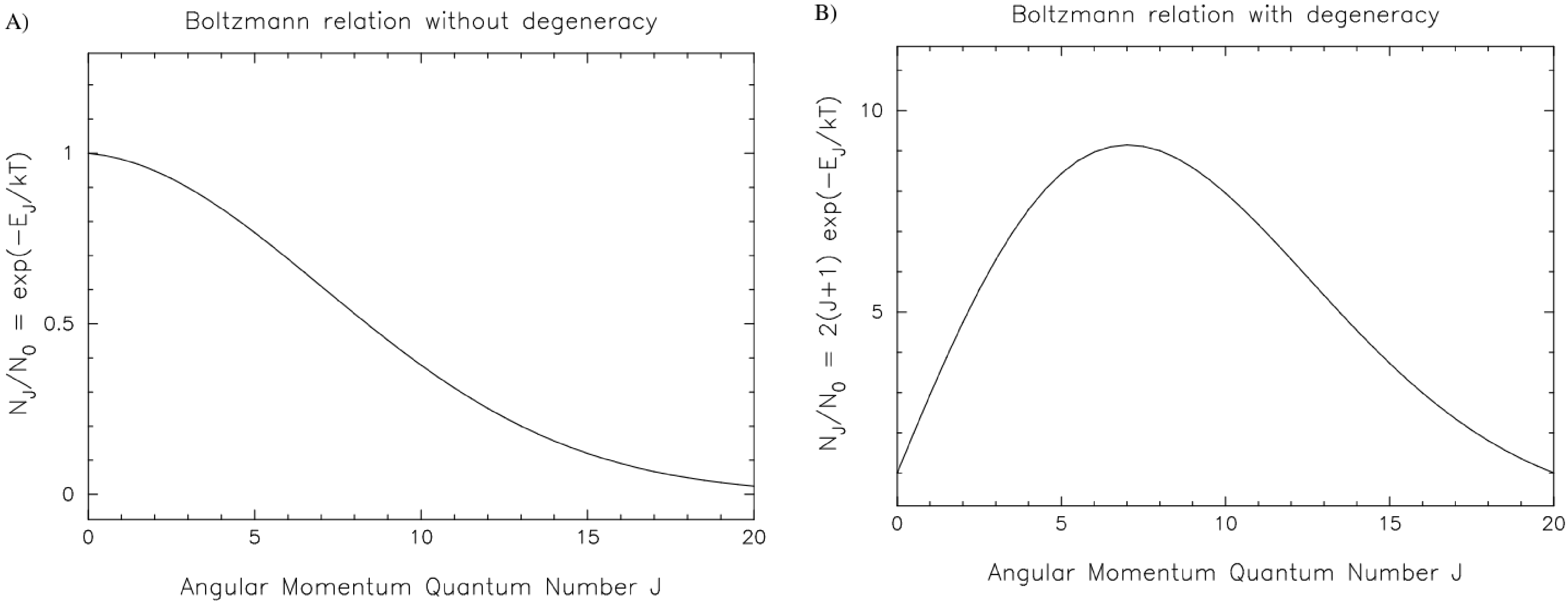

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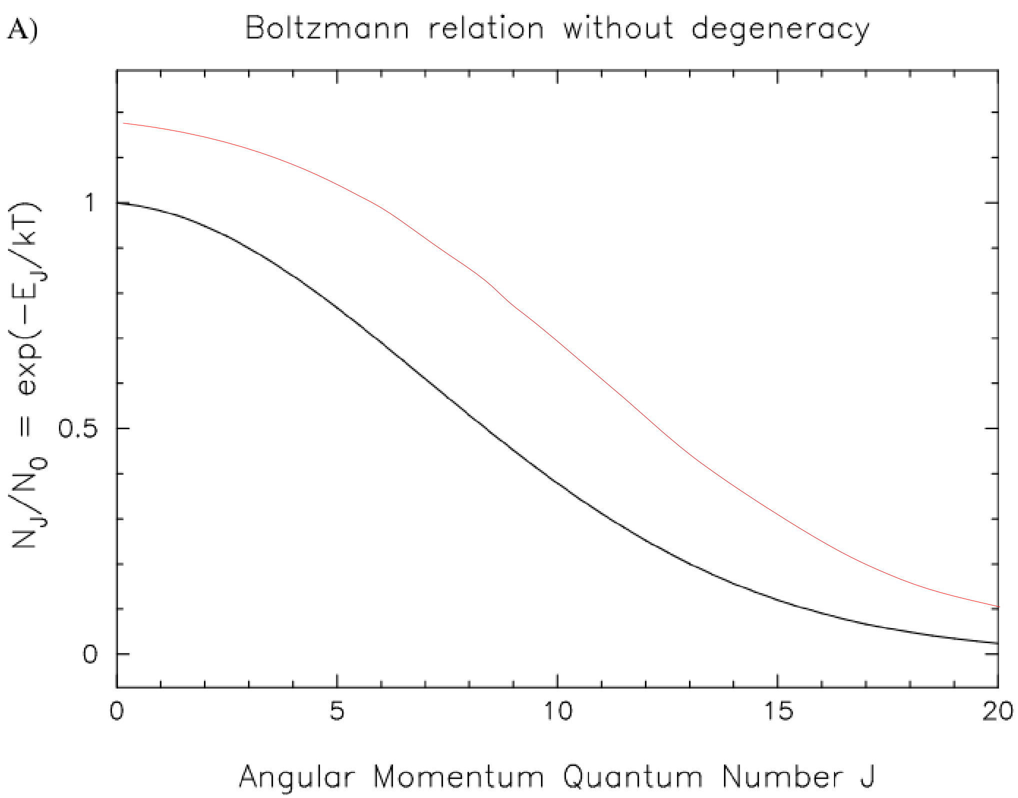

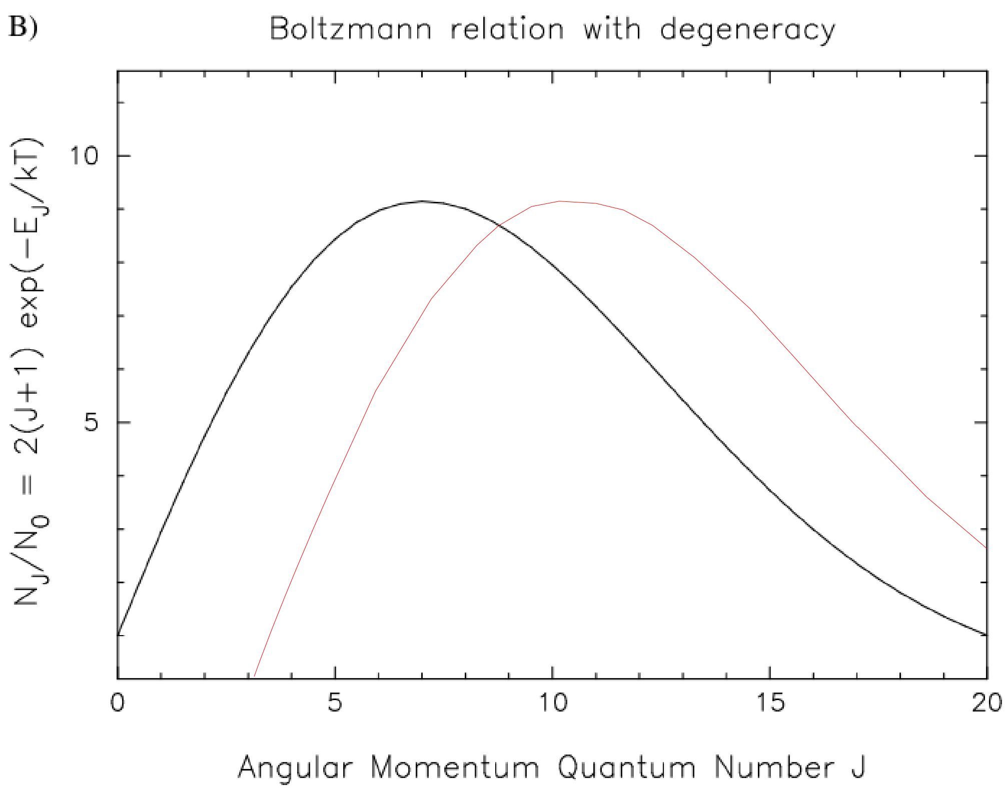

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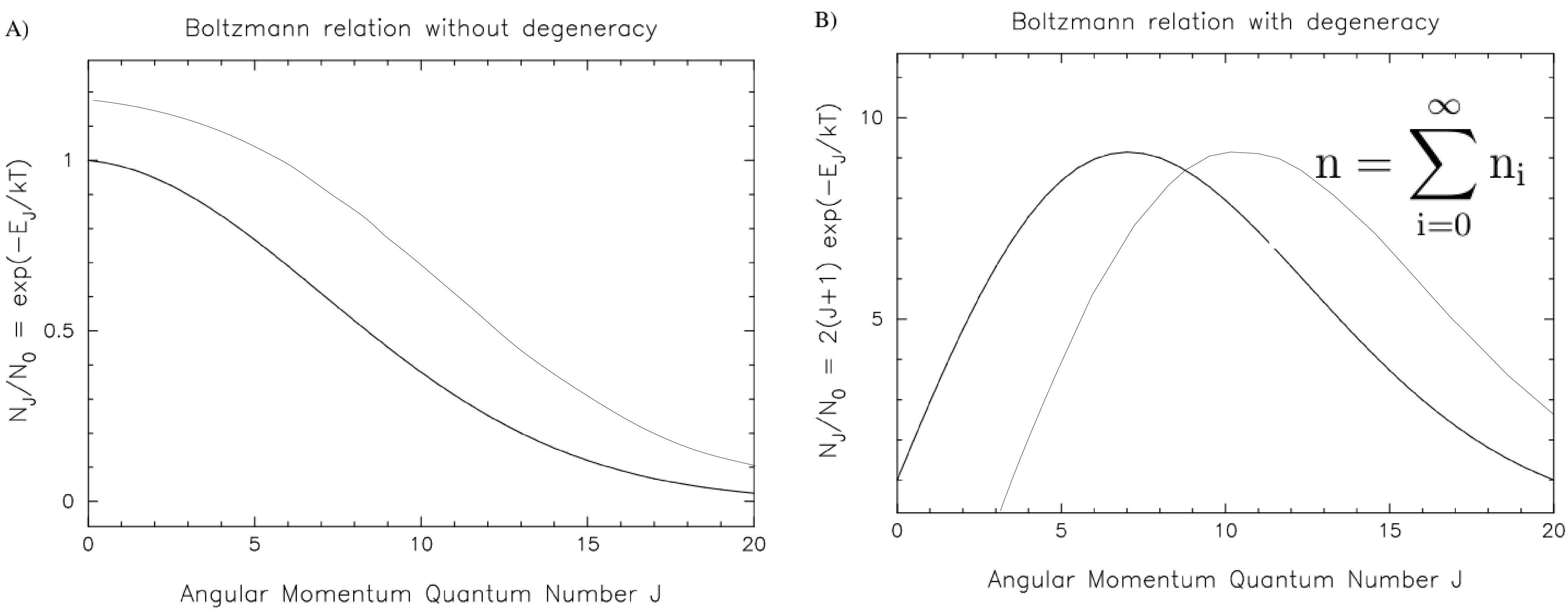

A little bit of Theory

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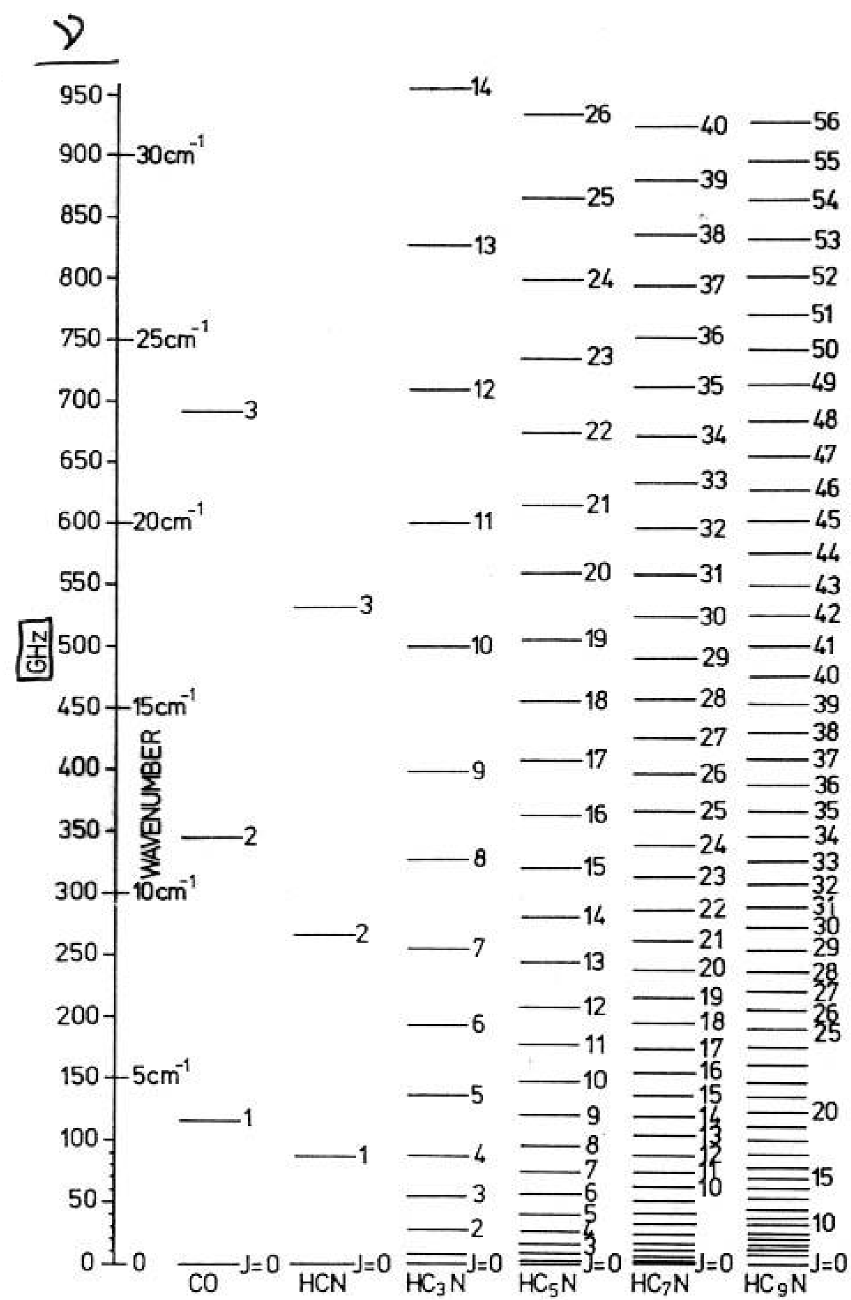
A little bit of Theory

Ensembles of rotating molecules

- The bigger the molecule, the larger the partition function
- At any given temperature the molecules can be distributed over a larger number of available energy levels



Slide – John Storey

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Dipole moment

A little bit of Theory

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Dipole moment	Number density	Degeneracy

- Full formula requires consideration of radiation transport

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THE UNIVERSITY OF SYDNEY

A little bit of Theory

Aside: Line profiles

- Gaussian line profiles often assumed for spectral lines

$$\phi(\nu) = \frac{\sqrt{4\ \ln\ 2}}{\Delta\nu\sqrt{\pi}}\ \mathrm{e}^{-4\ln 2\,(\frac{\nu}{\Delta\nu})^2}$$

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Turbulent Doppler broadening:

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A little bit of Theory

Molecular excitation and Einstein coefficients

- Consider an ensemble of molecules with 2 energy levels Eu and El
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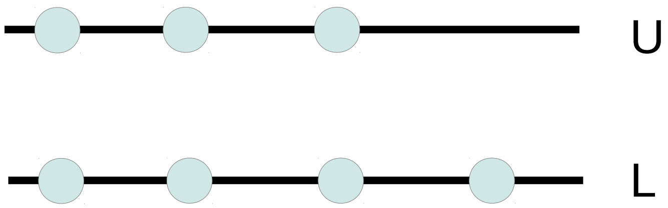

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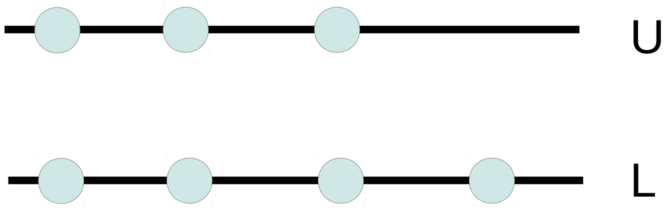

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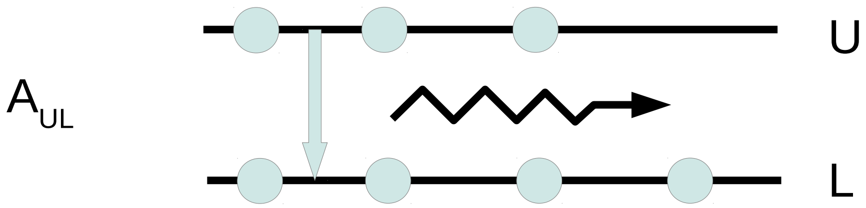

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A little bit of Theory

Molecular excitation and Einstein coefficients

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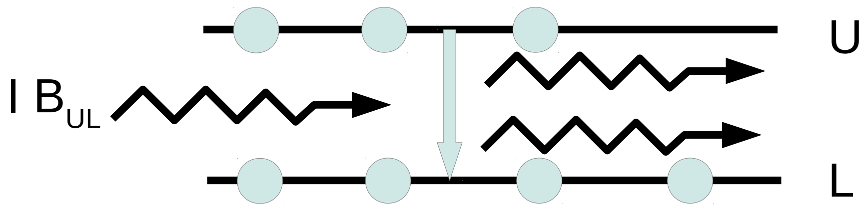

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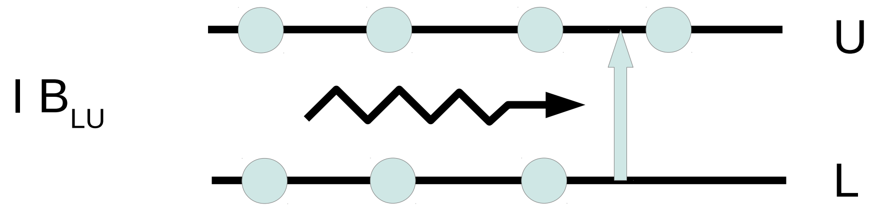

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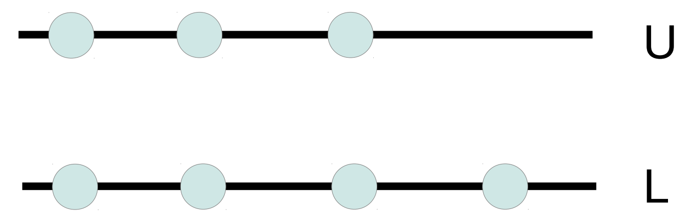

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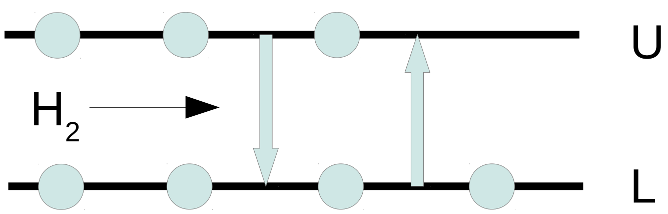

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- Collisions with H_2 also excite and de-excite levels:

$\mathrm{C_{lu}} = \mathrm{n_{H_2}}\,\gamma_{\mathrm{lu}}$ = Rate of collision induced transitions from lower to upper level

$\mathrm{C_{ul}} = \mathrm{n_{H_2}}\,\gamma_{\mathrm{ul}}$ = Rate of collision induced transitions from upper to lower level

A little bit of Theory

Molecular excitation and Einstein coefficients

- In the steady state the number of molecules in the levels remain constant

$$\mathrm{n_u}\left[\mathrm{A_{ul}} + \mathrm{B_{ul}}\,\bar{\mathrm{I}}_\nu + \mathrm{C_{ul}}\right] = \mathrm{n_l}\left[\mathrm{B_{lu}}\,\bar{\mathrm{I}}_\nu + \mathrm{C_{lu}}\right]$$

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A little bit of Theory

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- If the radiation originates from a blackbody (e.g., the CMB) with a temperature T_{bg}, the excitation temperature and kinetic temperature of a gas bathed in a background radiation field of temperature Tbg is given by:

$$e^{(E_u - E_l)/k\,T_{ex}} = \frac{A_{ul}[1 + J_\nu(T_{bg})] + C_{ul}}{A_{ul} J_\nu(T_{bg}) + C_{ul}\, e^{-E_u/k\,T_{kin}}}$$

A little bit of Theory

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- Two important cases: **collisions unimportant** or **collisions dominate**

A little bit of Theory

Molecular excitation and Einstein coefficients

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  Mostly radiative excitation and population is in equilibrium with background

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- Measure of the density at which collisional excitation becomes effective

A little bit of Theory

Radiative Transfer

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A little bit of Theory

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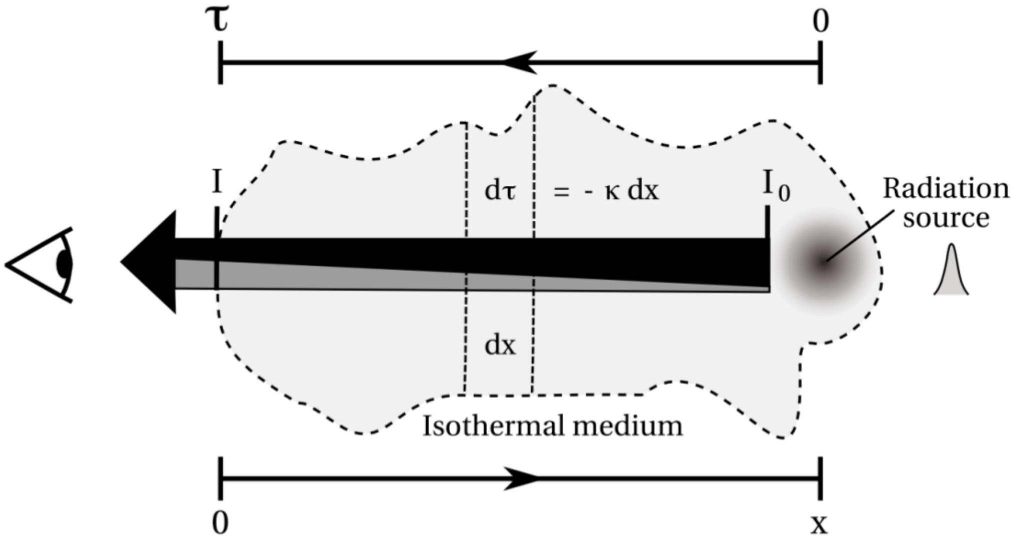
- In the real world the spectral lines are modified by the medium through which they pass – radiative transfer.



A little bit of Theory

Radiative Transfer

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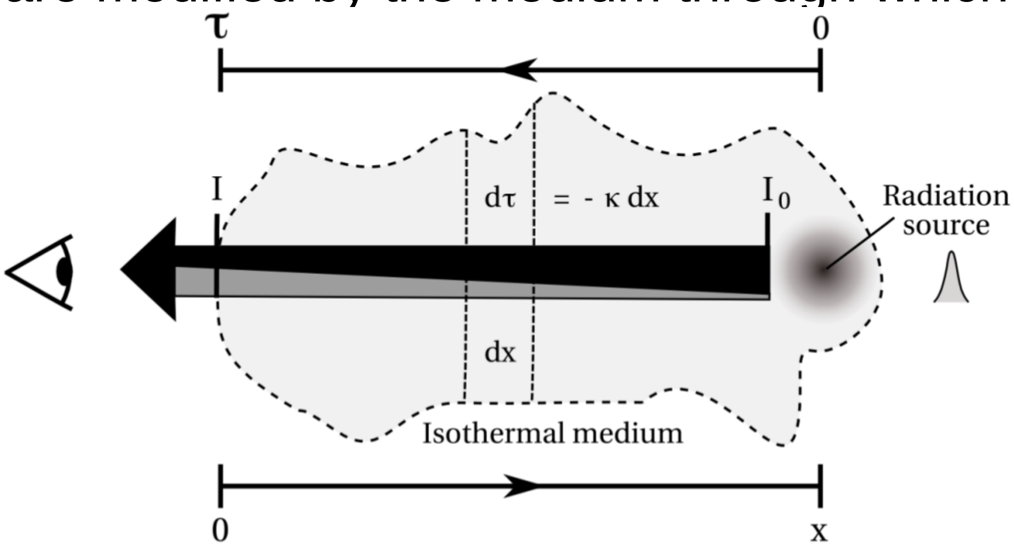
- Radiative Transfer Equation:

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + \epsilon_\nu$$



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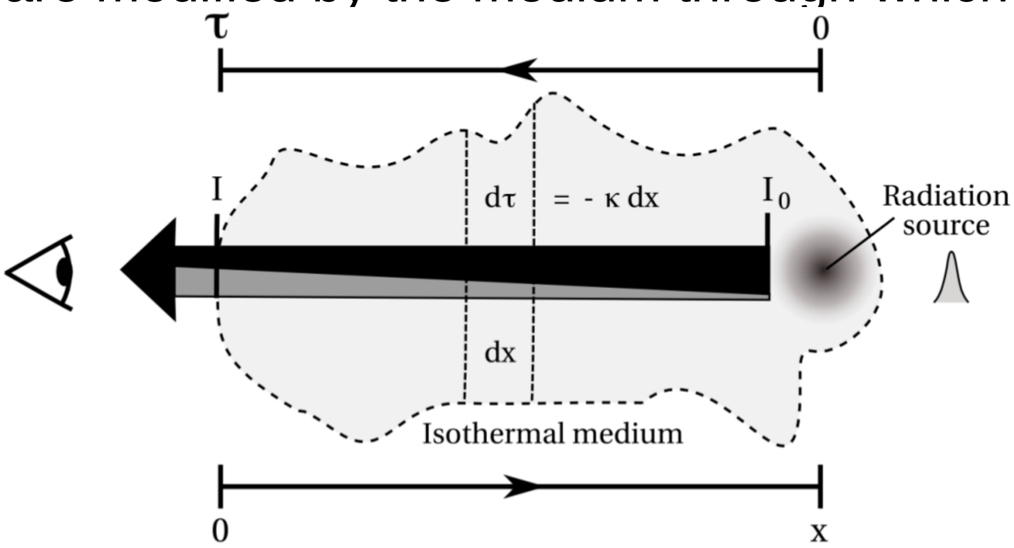


- Related to the molecular emission via the molecule's Einstein Coefficients:

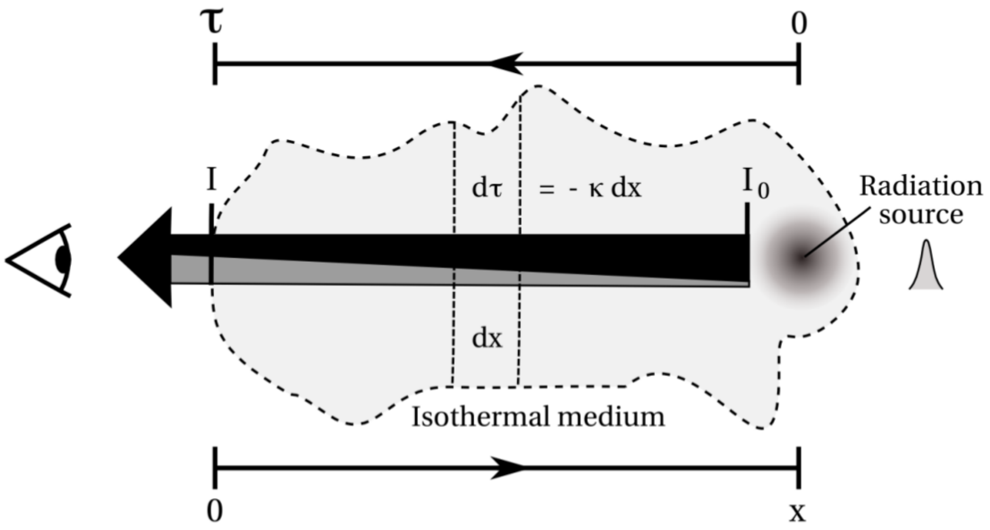
$$\epsilon_\nu = \frac{h\nu_{ul}}{4\pi} n_u A_{ul} \phi(\nu) \qquad \kappa_\nu = \frac{h\nu_{ul}}{4\pi}(n_l B_{lu} - n_u B_{ul})\phi(\nu)$$

A little bit of Theory

Radiative Transfer

- The definition of optical depth is useful:

$$\tau_\nu = -\kappa_\nu \mathrm{ds}$$



A little bit of Theory
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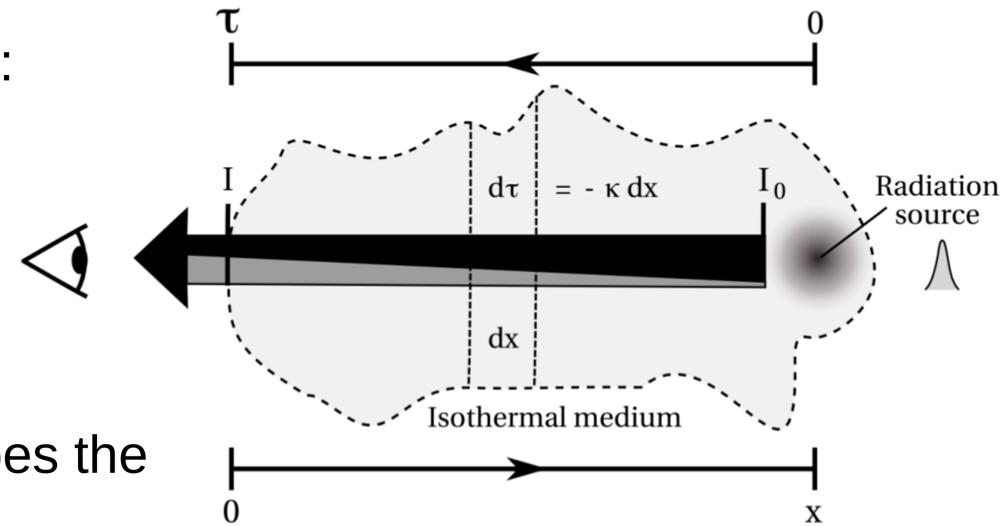
Where $\mathrm{S}_\nu = \epsilon_\nu / \kappa_\nu$ completely describes the medium and is known as the source function



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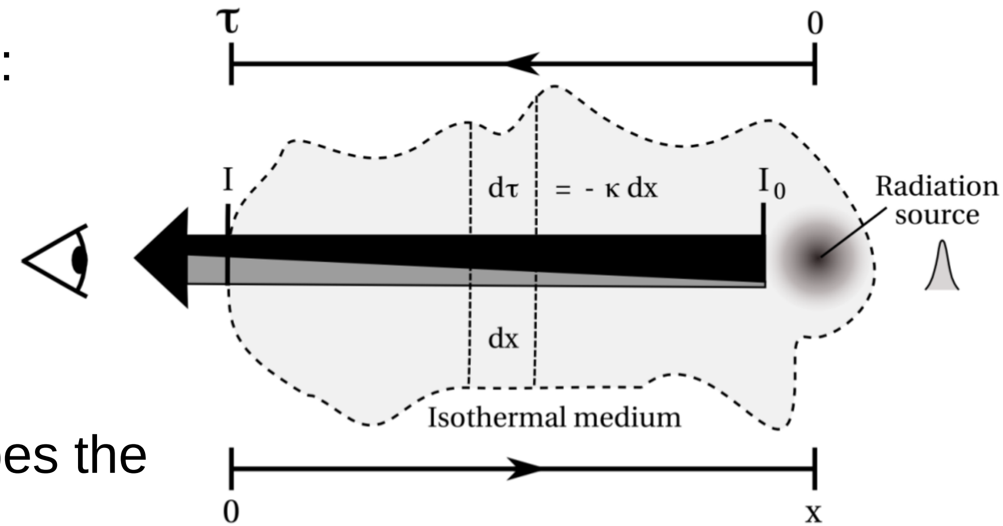

- Solving the radiative transfer equation we get:

$$\mathrm{I}_{\nu,\mathrm{s}} = \mathrm{I}_{\nu,0}\mathrm{e}^{-\tau_\nu} + \mathrm{S}_\nu(1 - \mathrm{e}^{-\tau_\nu})$$

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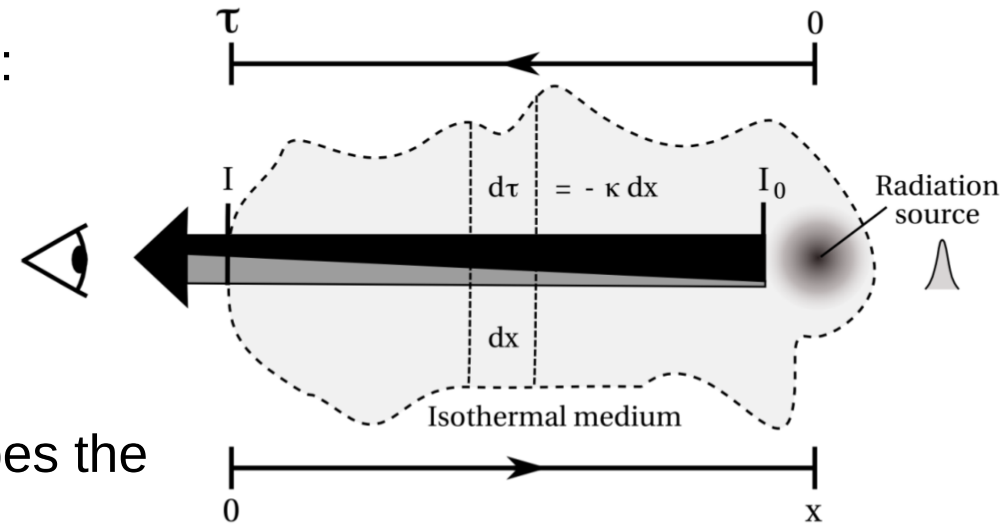

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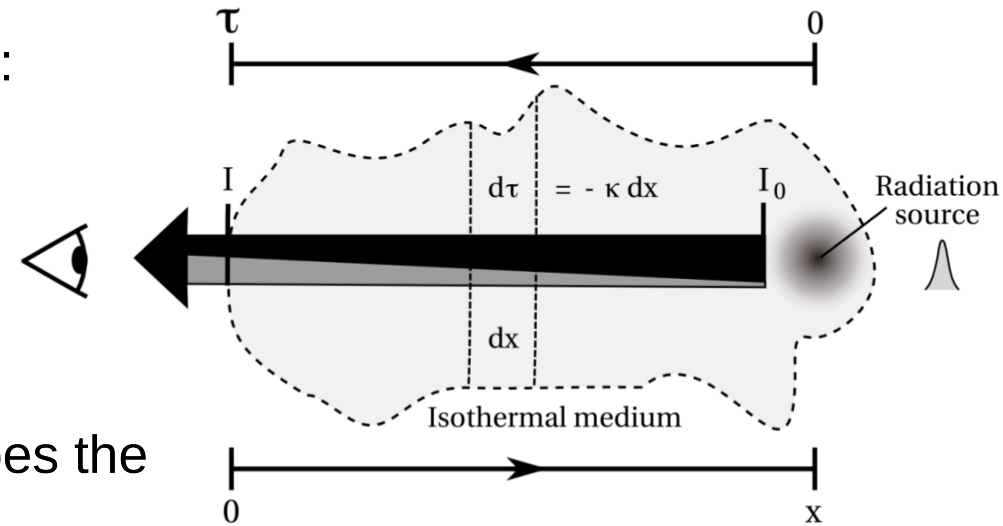

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Attenuated emission

Emitting medium

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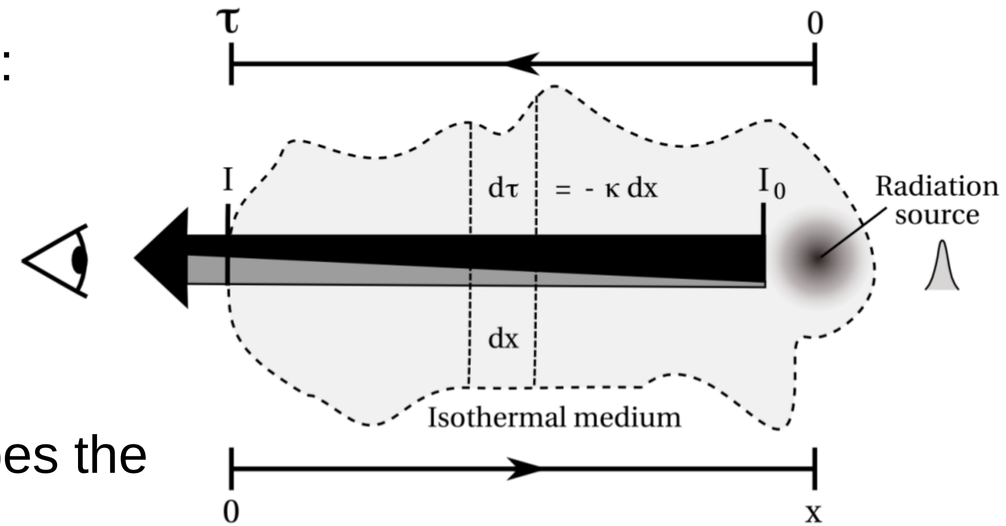

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Attenuated emission | Emitting medium

General solution assuming an isothermal homogeneous medium

A little bit of Theory

Radiative Transfer

- Brightness temperature is defined as the temperature measured of the source function was well approximated by the Rayleigh-Jeans law:

$$T_b = \frac{c^2}{2k\nu^2} B_\nu(T_R) = \frac{h\nu}{k} J_\nu(T_R)$$

$$J_\nu(T) = (e^{(E_u - El)/kT} - 1)^{-1}$$

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- Two special cases:

Optically Thin Emission: $(\tau \ll 1)$ $$T_b = \frac{h\nu}{k} [J_\nu(T_s) - J_\nu(T_{bg})]\tau_\nu$$

Optically Thick Emission: $(\tau \gg 1)$ $$T_b = \frac{h\nu}{k} [J_\nu(T_s) - J_\nu(T_{bg})]$$

A little bit of Theory

Physical parameters from observations

- **Optically thick transition:**

  Brightness temperature of a line saturates at Tex

  Under LTE conditions Tkin = Tex

$$T_{\rm kin} = T_{\rm s} = \frac{h\nu}{k}\left[\ln\left(1 + \frac{(h\nu/k)}{T_{\rm b} + \frac{h\nu}{k} J_\nu(T_{\rm bg})}\right)\right]^{-1}$$

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- **Optically thin transition:**

  Intensity under a line proportional to Tex and the number of molecules

$$N_u = \frac{8k\pi\nu^2}{A_{ul}hc^3}\int_{-\infty}^{\infty} T_b\, dv\left(\frac{\tau_\nu}{1 - e^{-\tau_\nu}}\right)$$

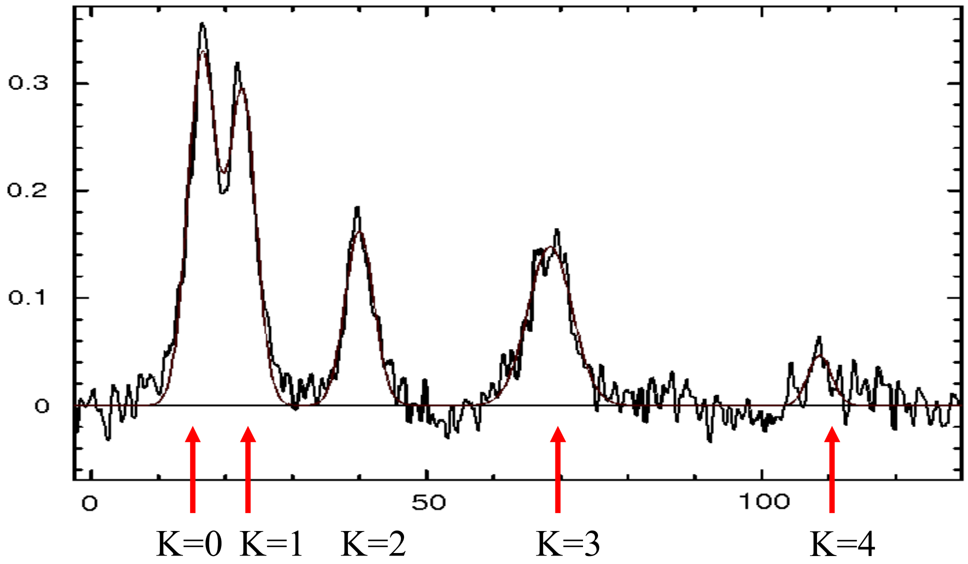
$$N = \frac{N_u}{g_u} e^{E_u/kT} Q(T_{ex})$$

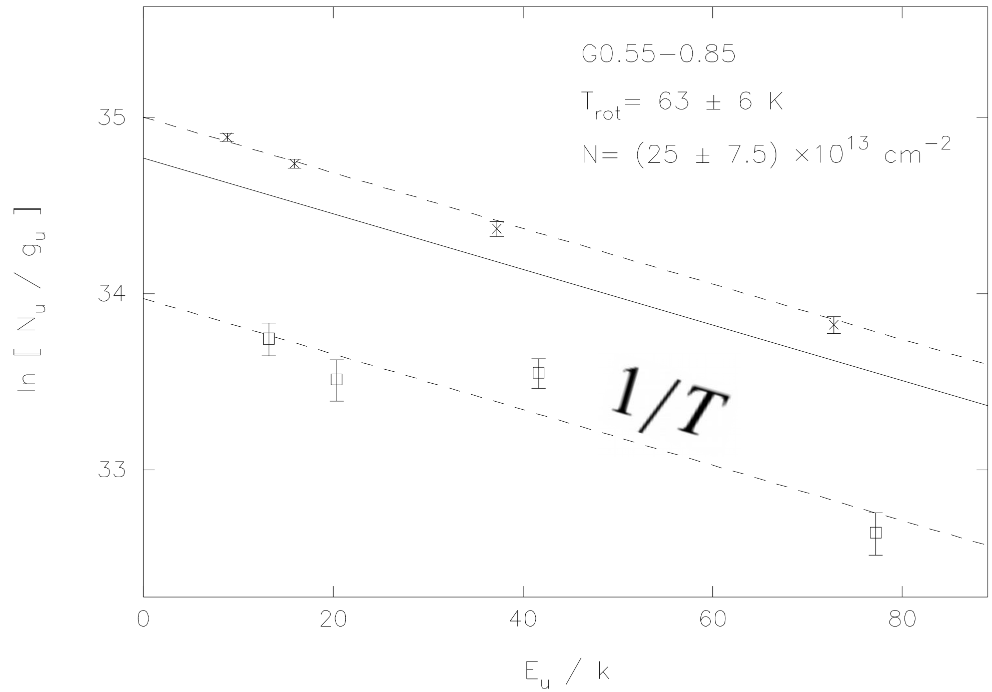
$$Q(T_{ex}) = \sum_i g_i\, e^{-E_i/kT_{ex}}$$

A little bit of Theory
Physical parameters from observations

- The rotation diagram

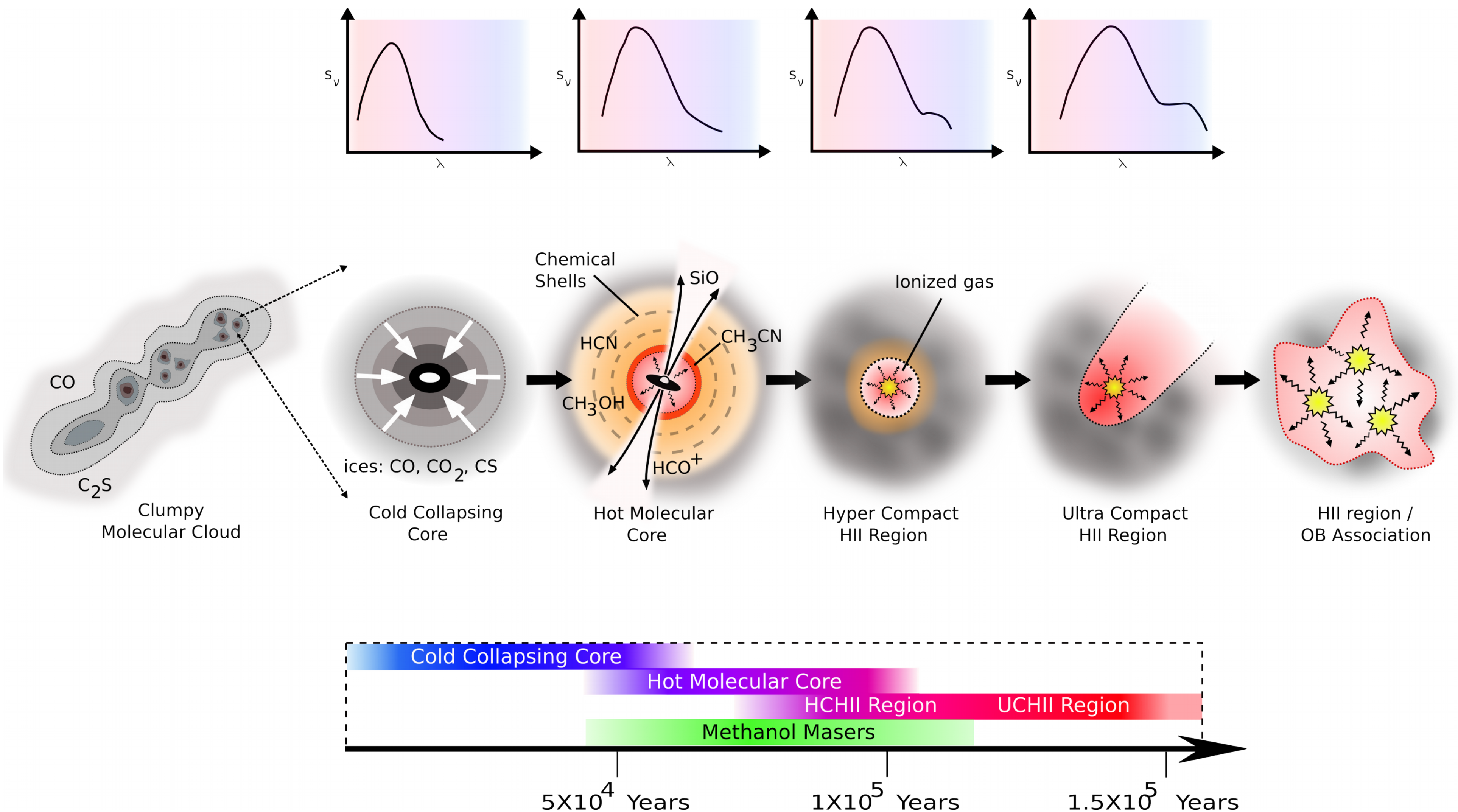
$$\ln\left(\frac{N_u}{g_u}\right) = \ln\left(\frac{N}{Q(T)}\right) - \frac{E_u}{kT_{ex}}$$





Case Study

Hot Molecular Cores



Case Study

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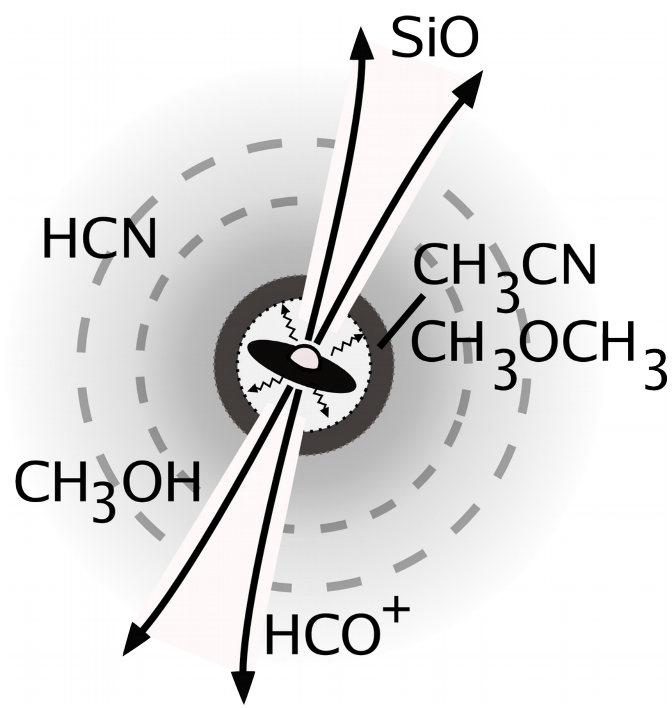

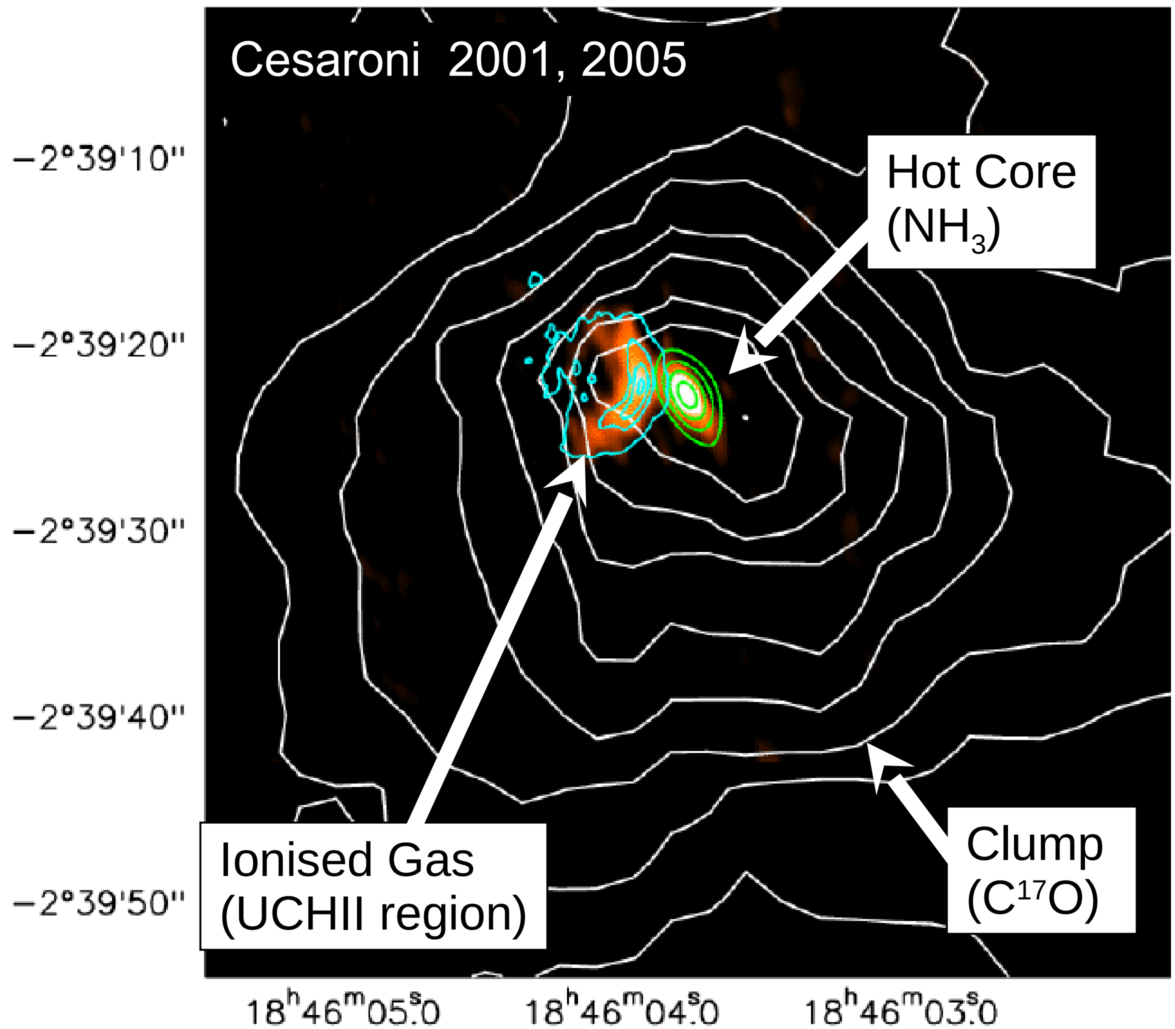

Case Study

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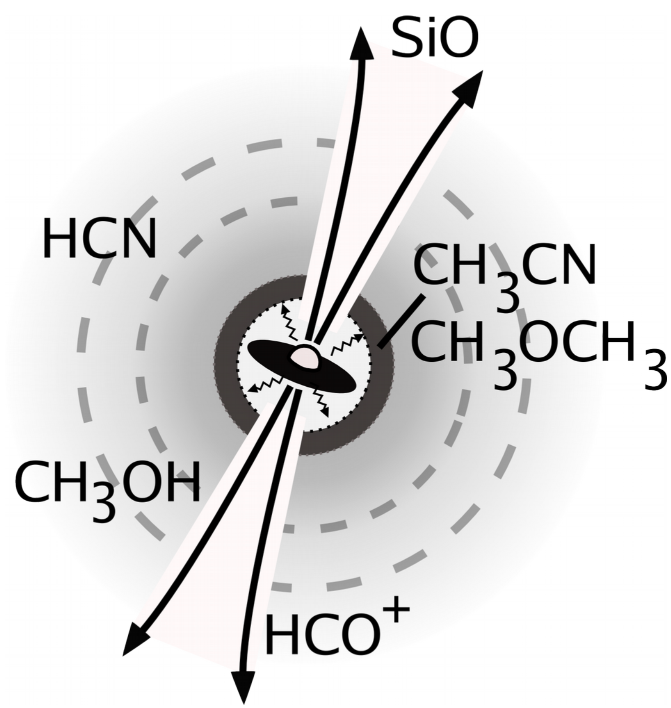
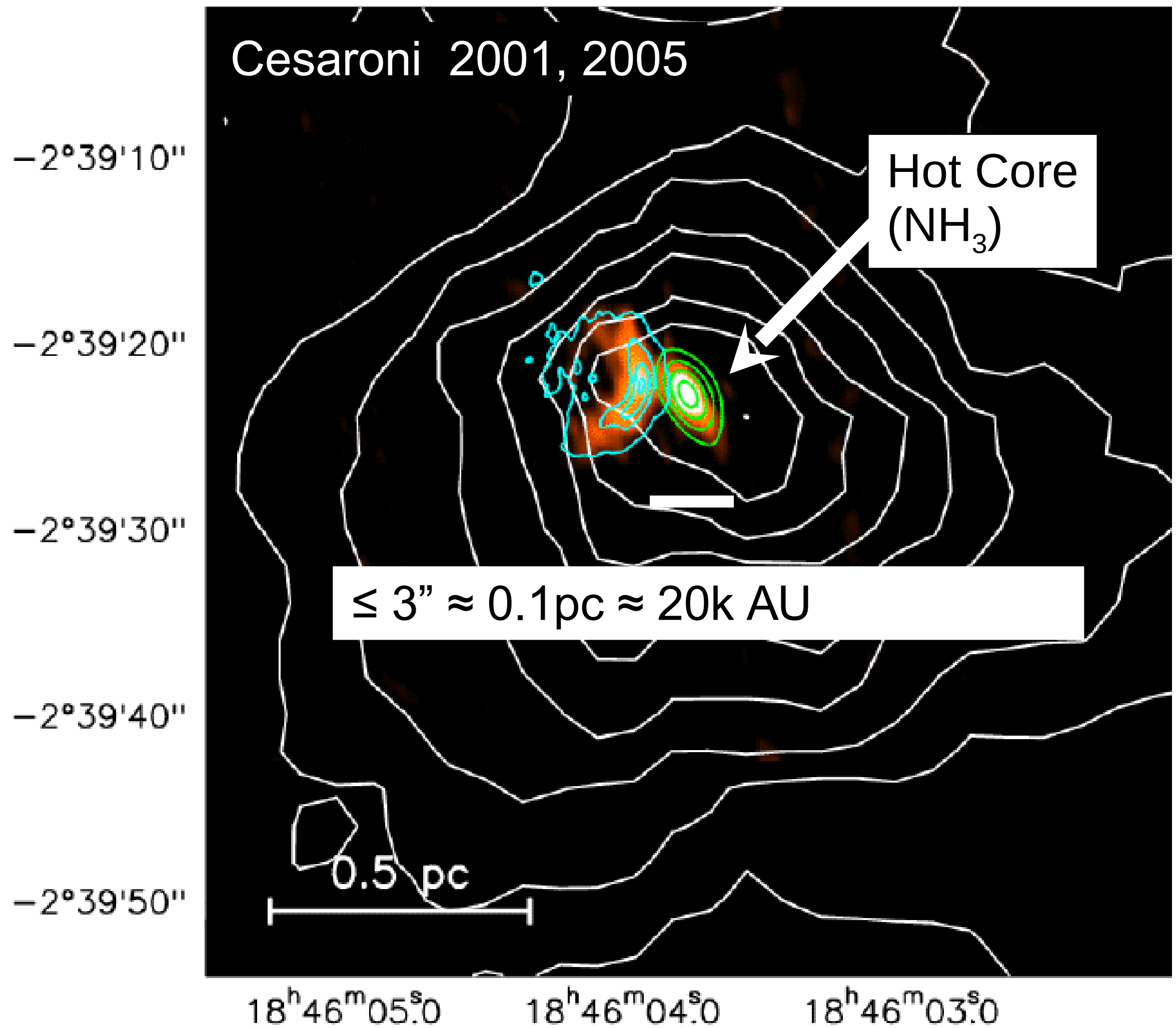
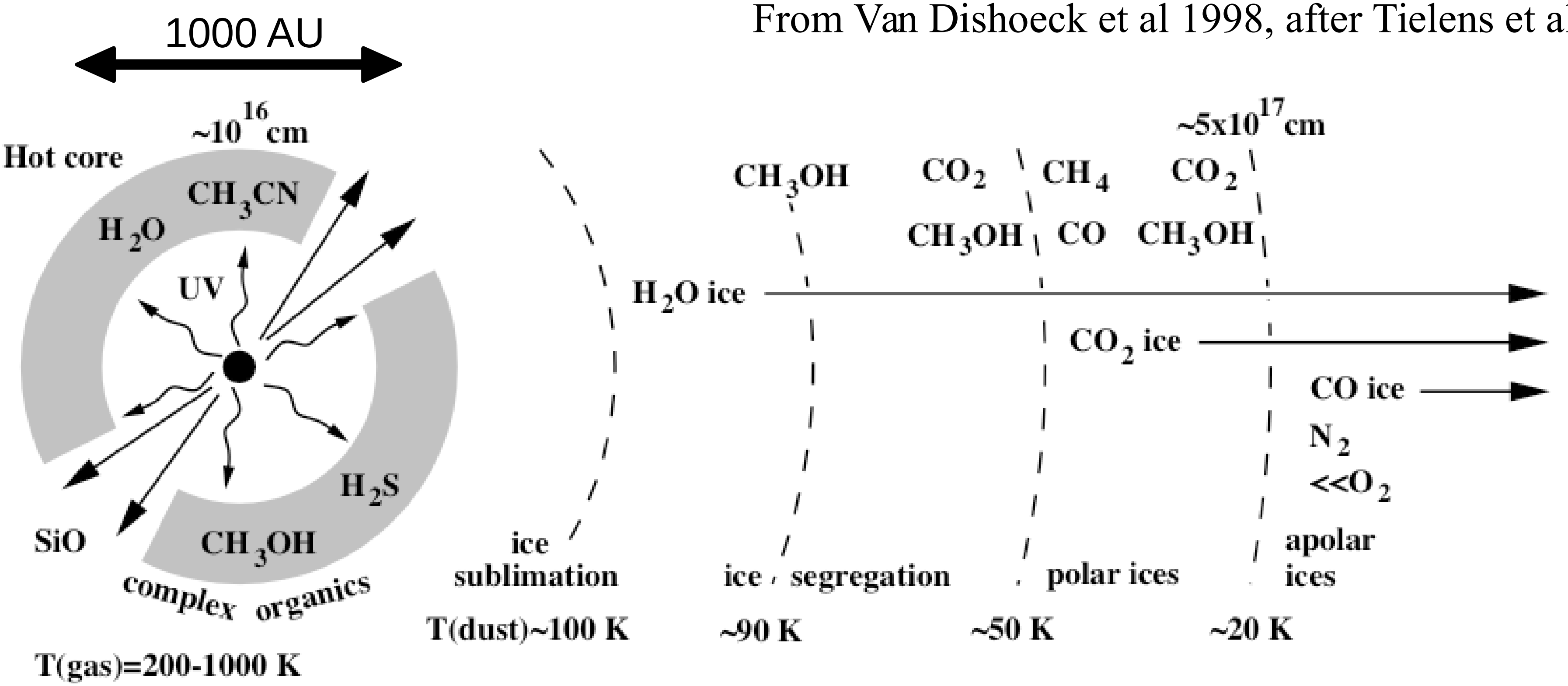




Case Study

Hot Molecular Cores - Chemistry

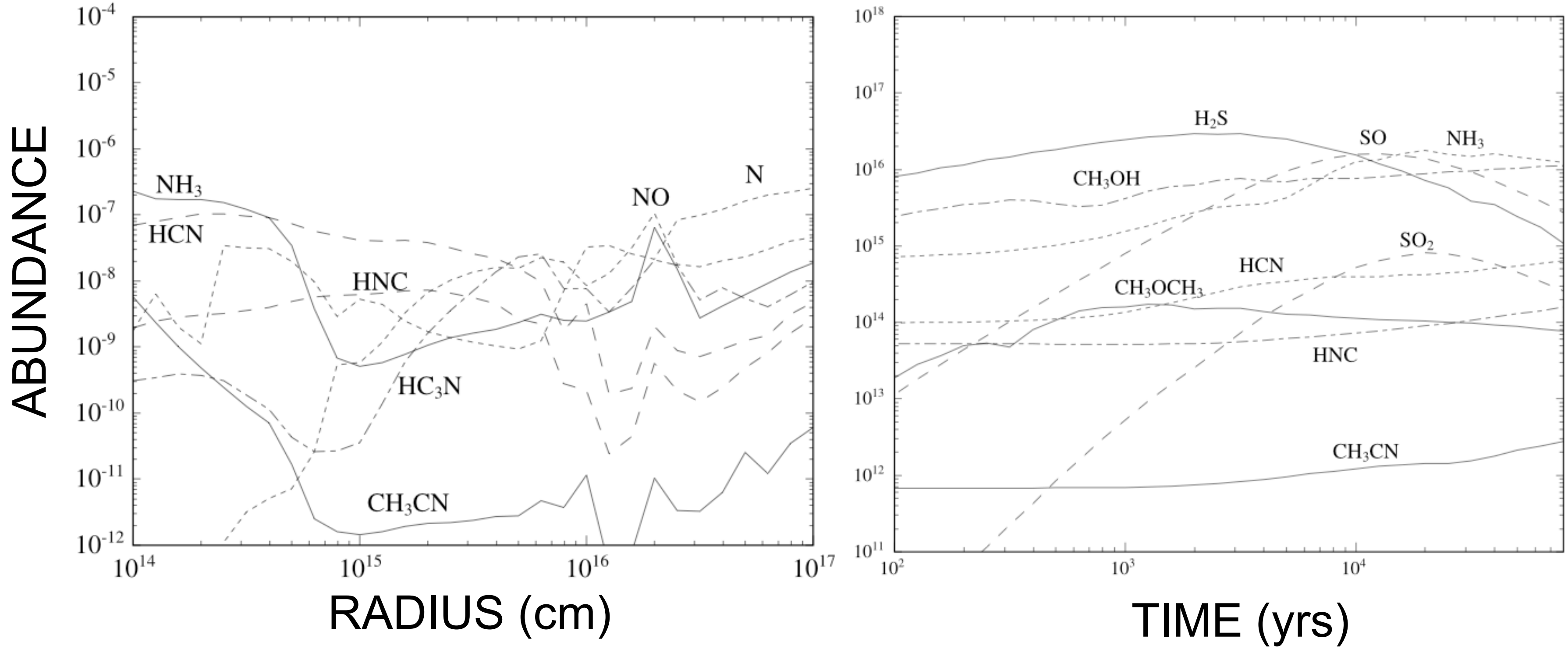


From Van Dishoeck et al 1998, after Tielens et al 1991

- Temperature gradient leads to an 'onion-layer' effect.
- Volatile non-polar ices evaporate at lower T, creating chemical shells.

Case Study
Hot Molecular Cores - Chemistry

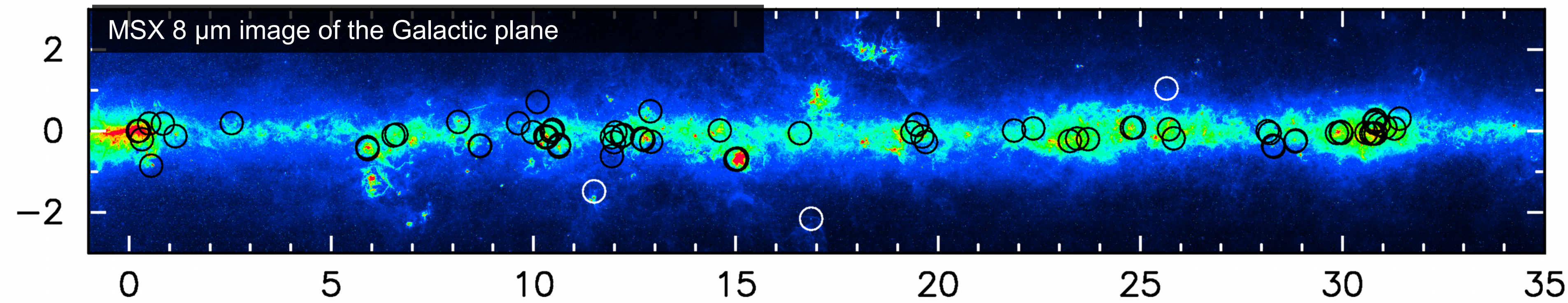
- However chemistry is also time-dependant as central object is evolving

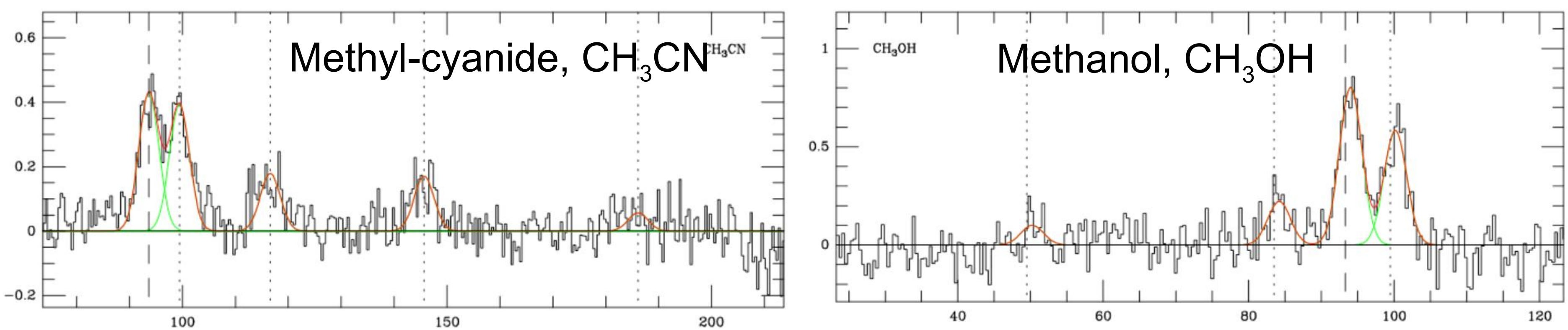


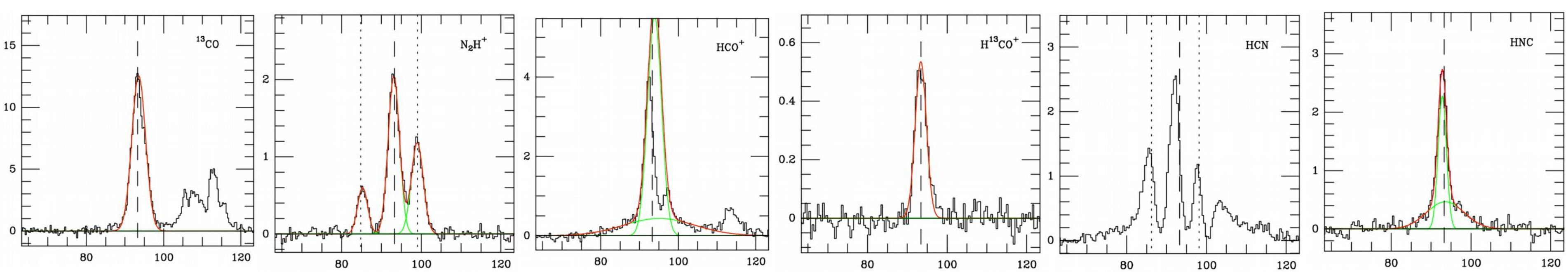
- Variables: initial abundances, geometry, mass, presence of shocks etc

Case Study

Hot Molecular Cores - Chemistry

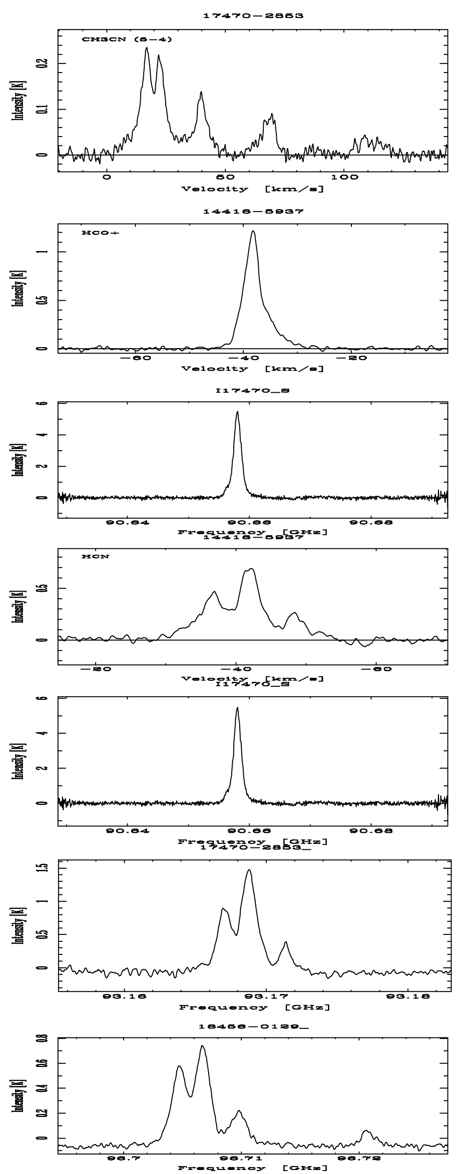






Case Study

Hot Molecular Cores - Chemistry



Molecule:	Usage:
CH_3CN (5-4) & (6-5)	Rotational Temperatures, Column Density Rich-chemistry tracer.
HCO^+ (1-0)	Signatures or outfall & inflow, sensitive to optical depth.
$H^{13}CO^+$ (1-0)	
HCN (1-0)	Ratio dependant on gas temperature, probes outer envelope.
HNC (1-0)	
N_2H^+ (1-0)	Excellent dense gas tracer- probe of central core.
CH_3OH (1-0)	Temperature probe, abundance vs # maser spots.

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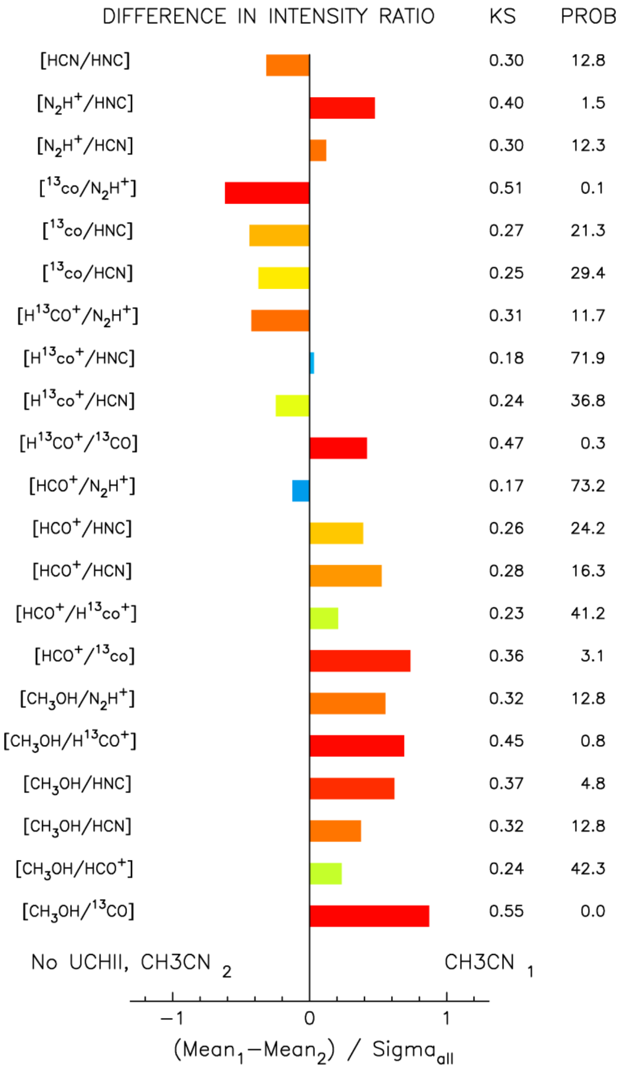
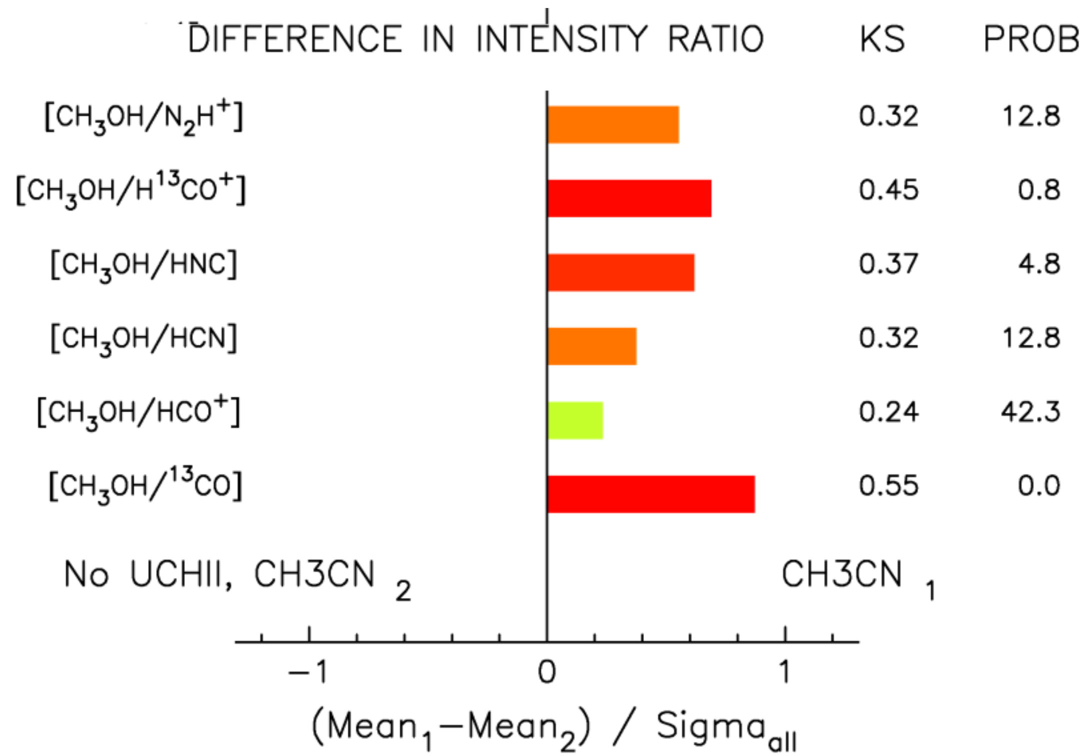
Case Study

Hot Molecular Cores - Chemistry



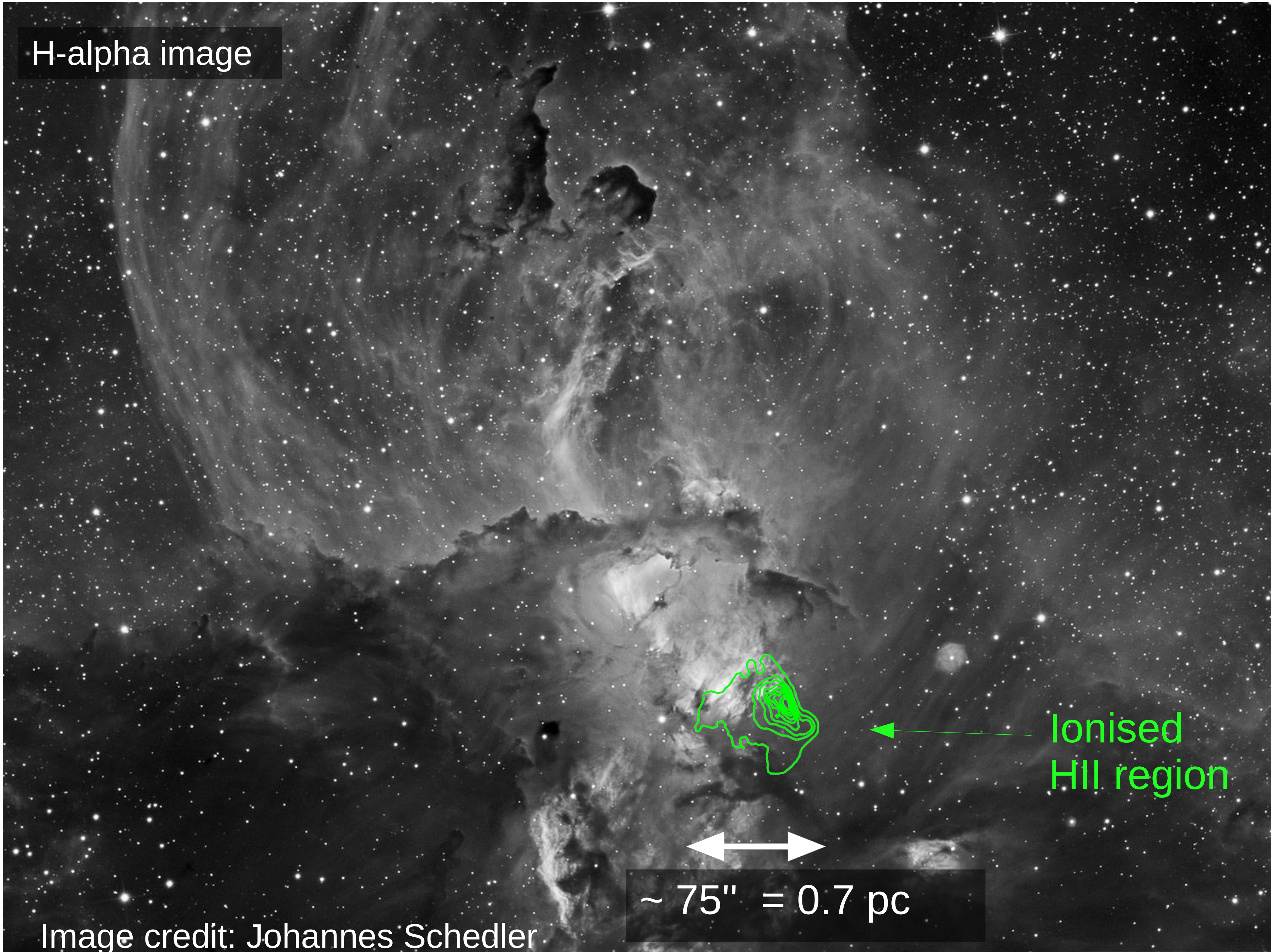


Case Study

Dissecting a star-forming region

Case Study

Dissecting a star-forming region



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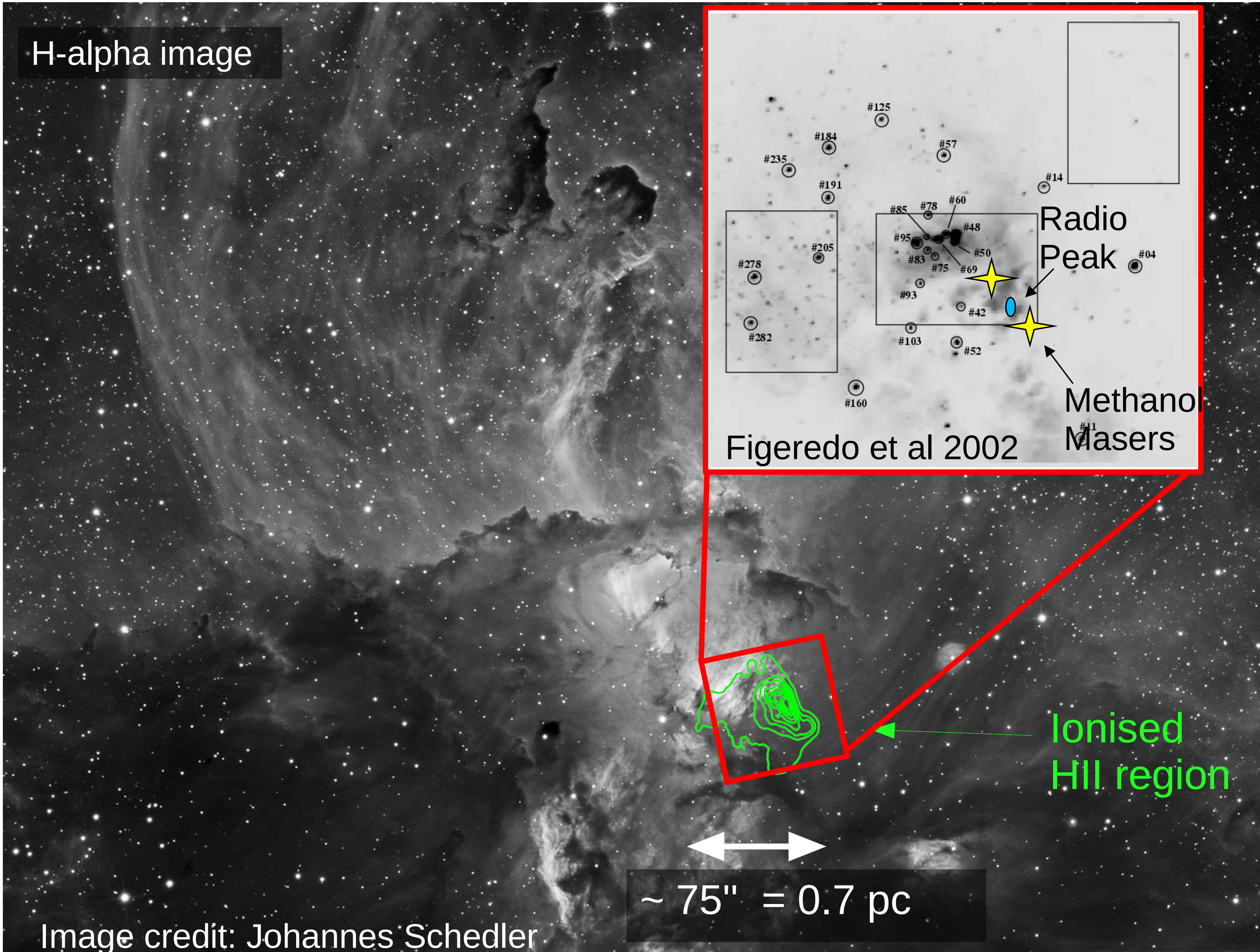

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Dissecting a star-forming region

H-alpha image

Sequential star formation?
- 42 sources with IR excess.
- NE-SW reddened colour gradient implies recent sequential SF

Ionised gas:
- Peaked & confined in west
- Electron temperature gradient

Other tracers of star-formation:
- Methanol Masers, Water Masers
- CO band-head (disks, winds?)

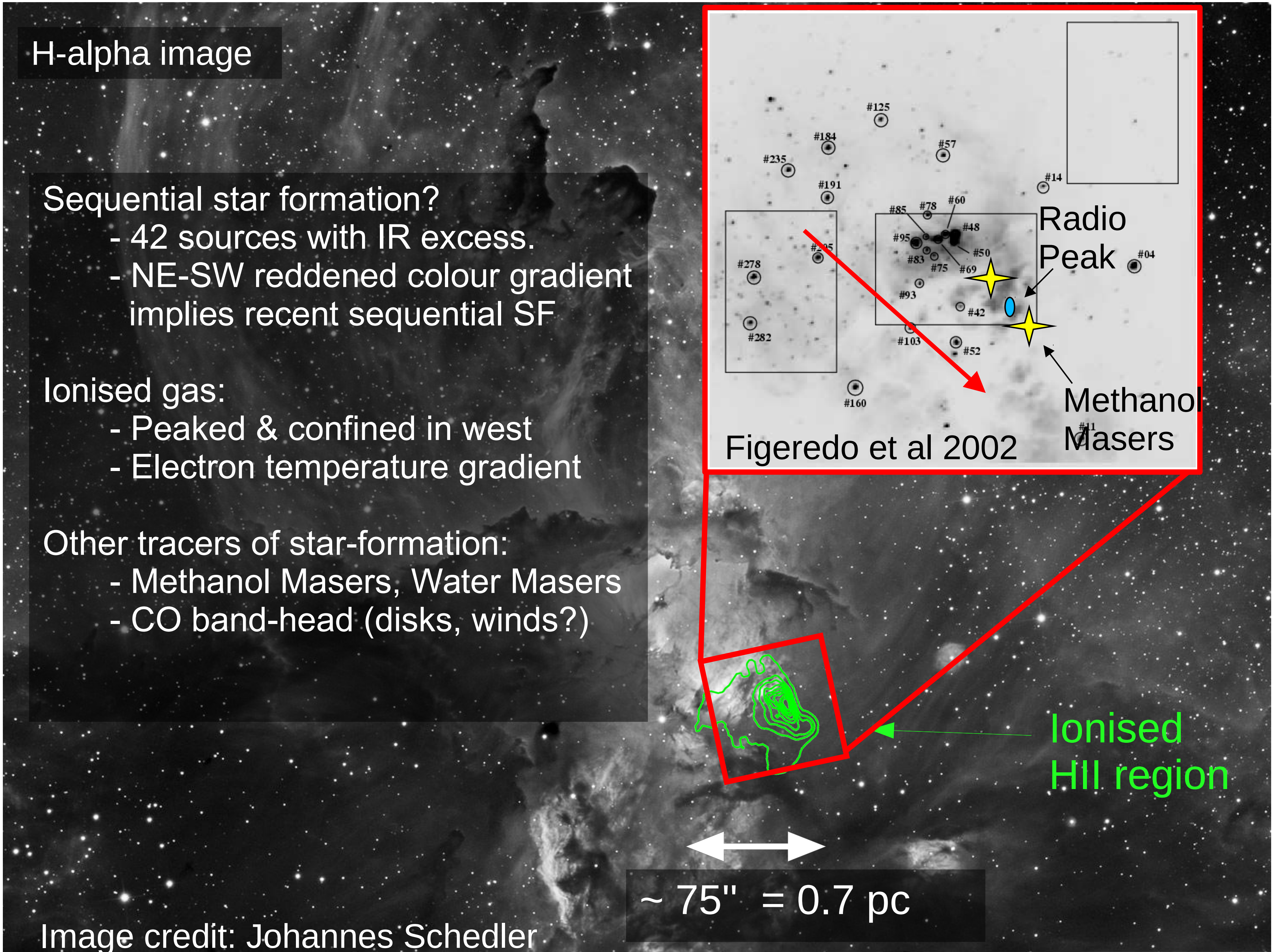

Image credit: Johannes Schedler

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Dissecting a star-forming region

Hill et al observed the region as part of a large 1.2-mm continuum survey
- Hill et al 2005, 2006

Sensitive to cool dust & free-free emission

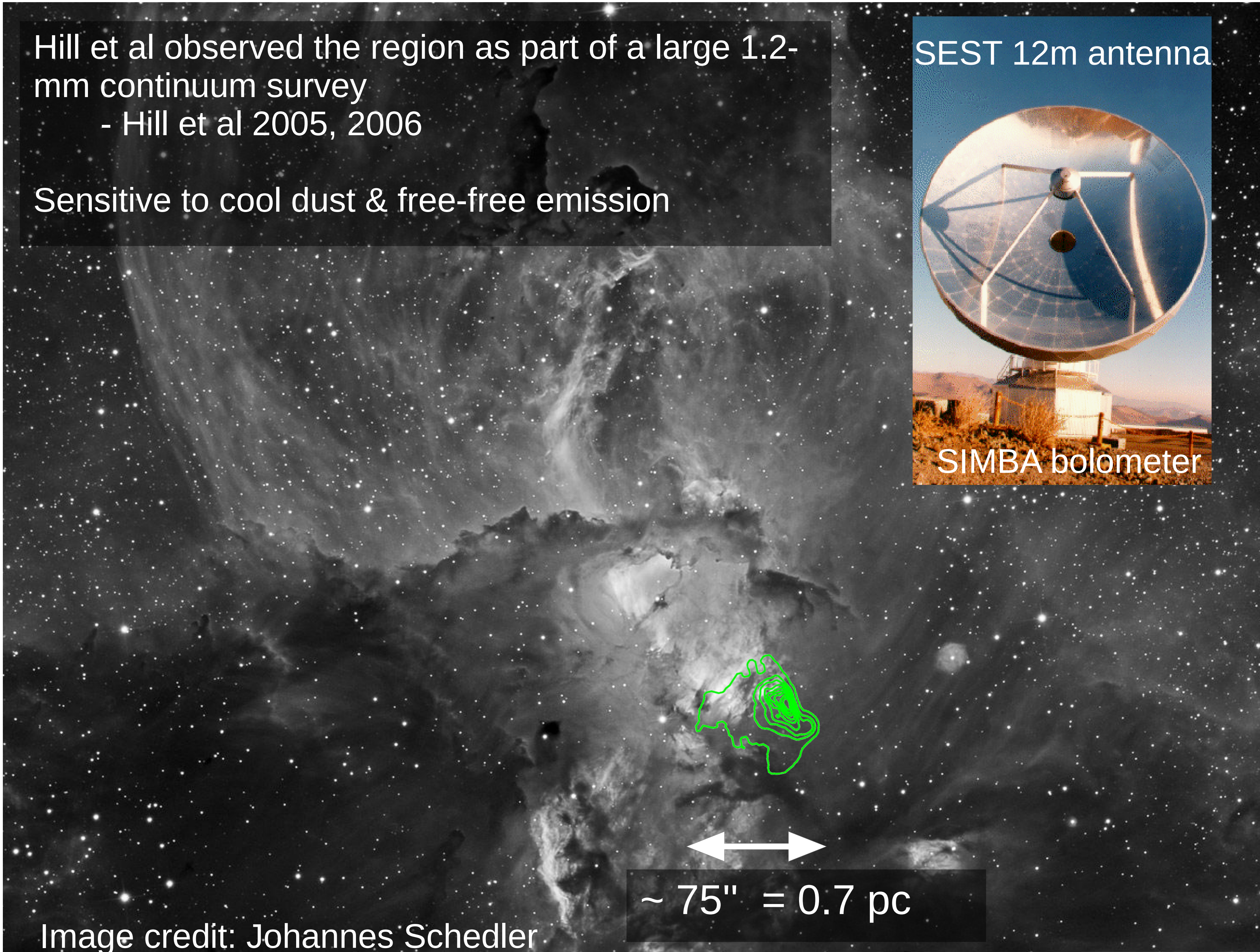

Image credit: Johannes Schedler

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Dissecting a star-forming region

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Sensitive to cool dust & free-free emission

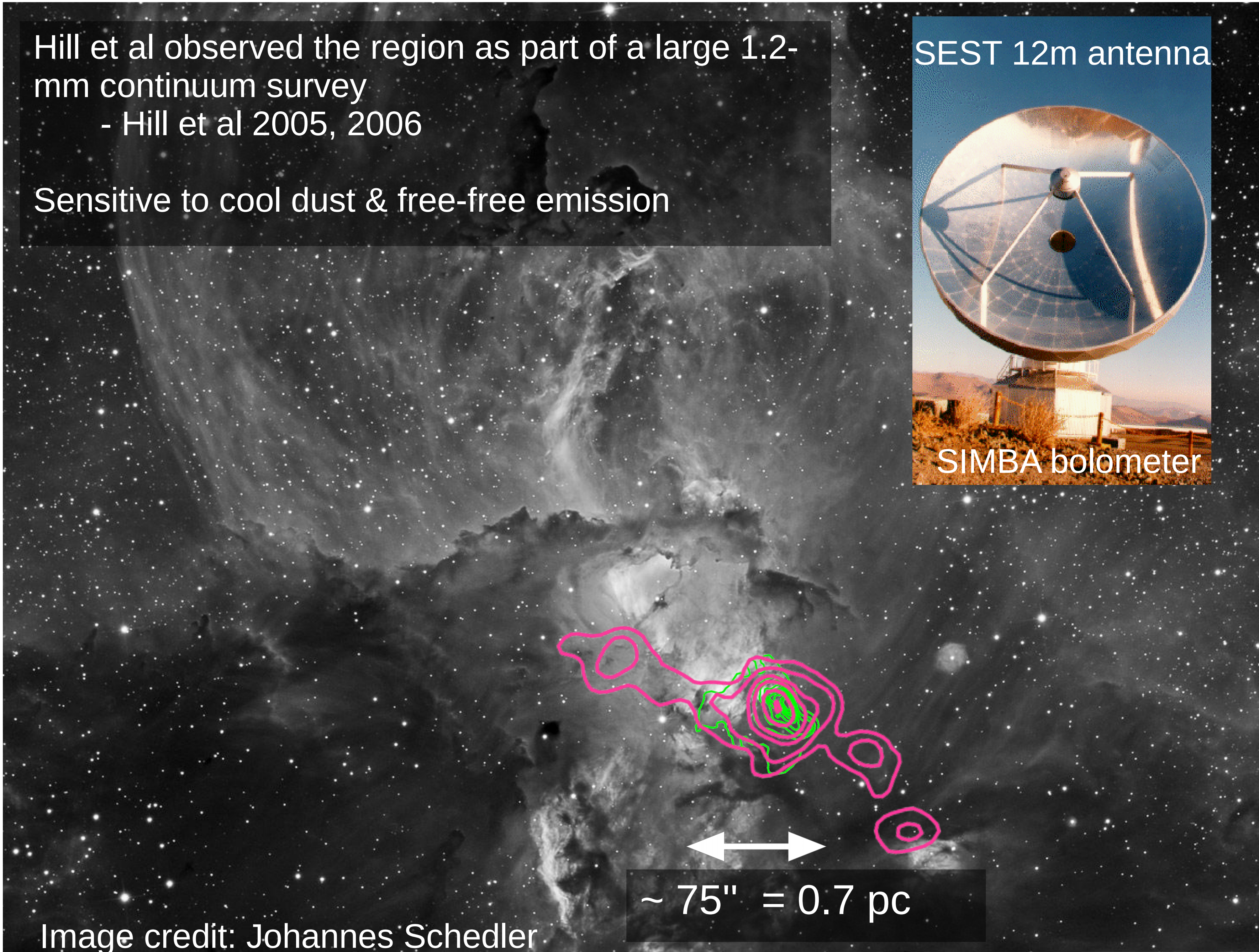

Image credit: Johannes Schedler

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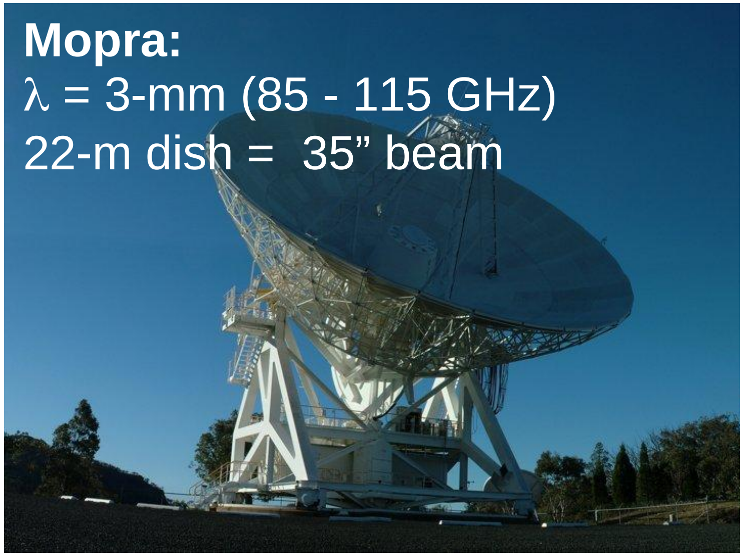

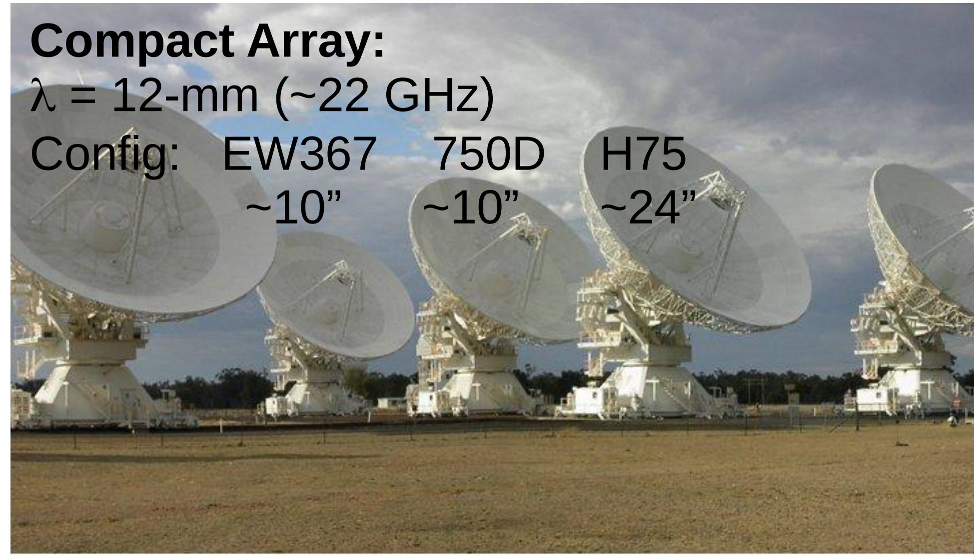

Mopra:

CO … diffuse gas

HCO^+ … kinematics

CS … intermediate gas

N_2H^+ … dense gas

Compact Array:

Ammonia (NH_3) … thermometer

H_2O masers … kinematics

23 GHz continuum
- Ionised gas
- Ultra-compact HII regions
- Hyper-compact HII regions

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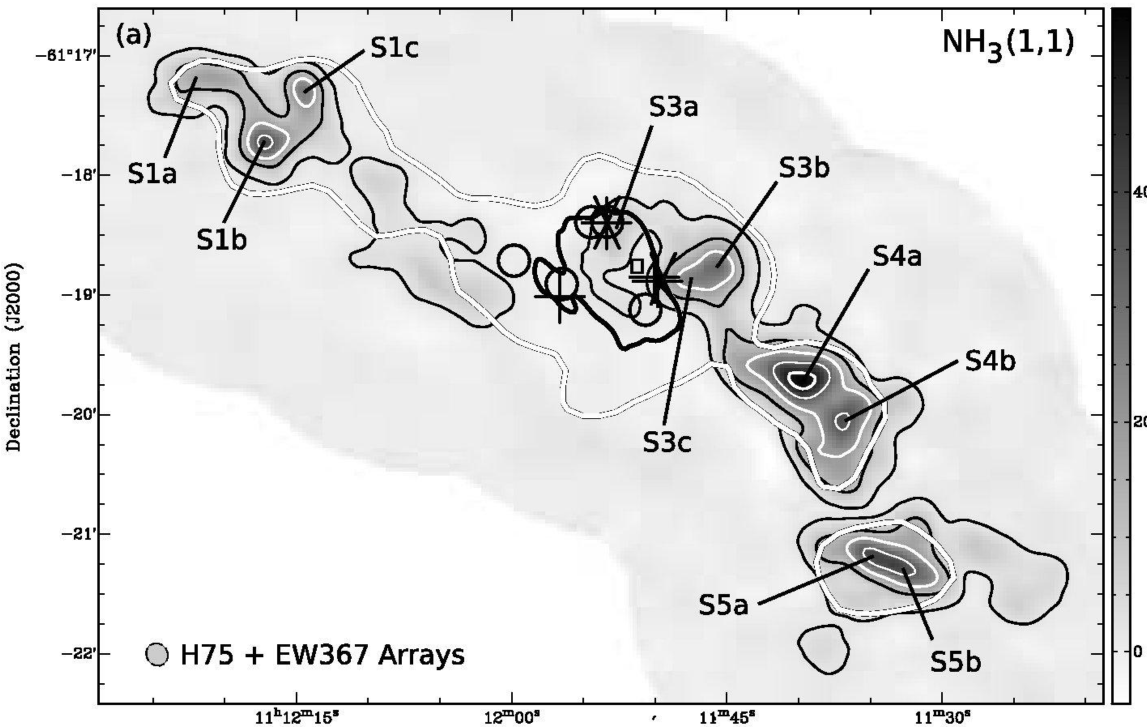
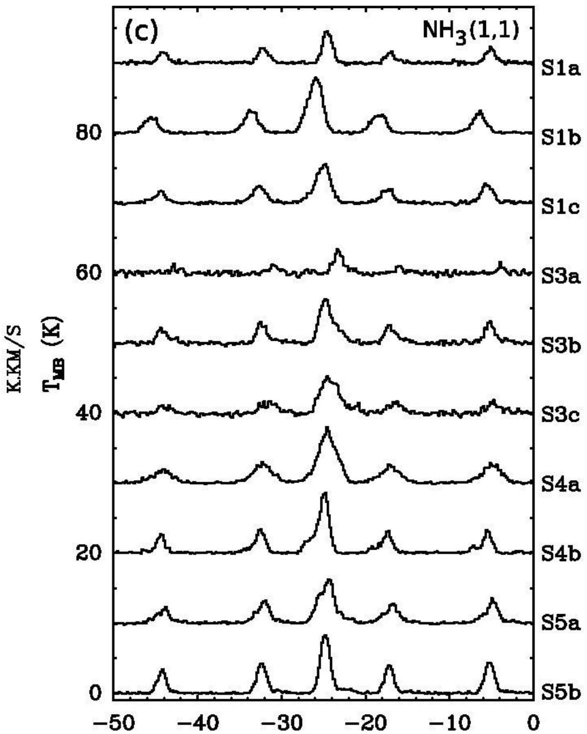
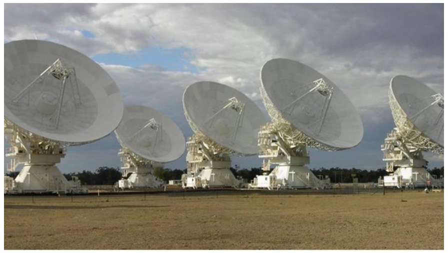
Dissecting a star-forming region





White Contours = 1.2mm emission
Thick Black Contour = 23 GHz continuum
Beam FWHM ~ 10''



- NH_3 emission follows 1.2-mm (except in HII region)
- Resolves 'clumps' (~0.5pc) into 'cores' (~0.1pc).

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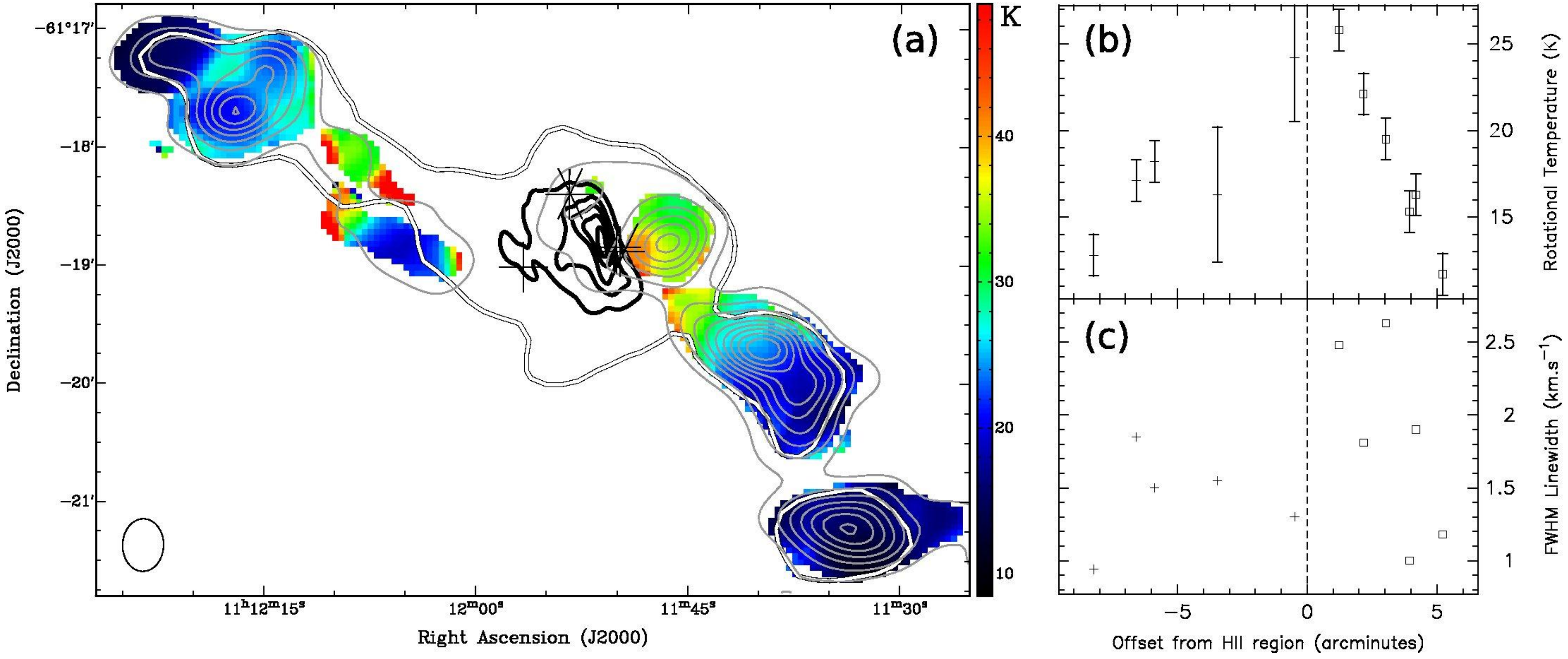
Dissecting a star-forming region



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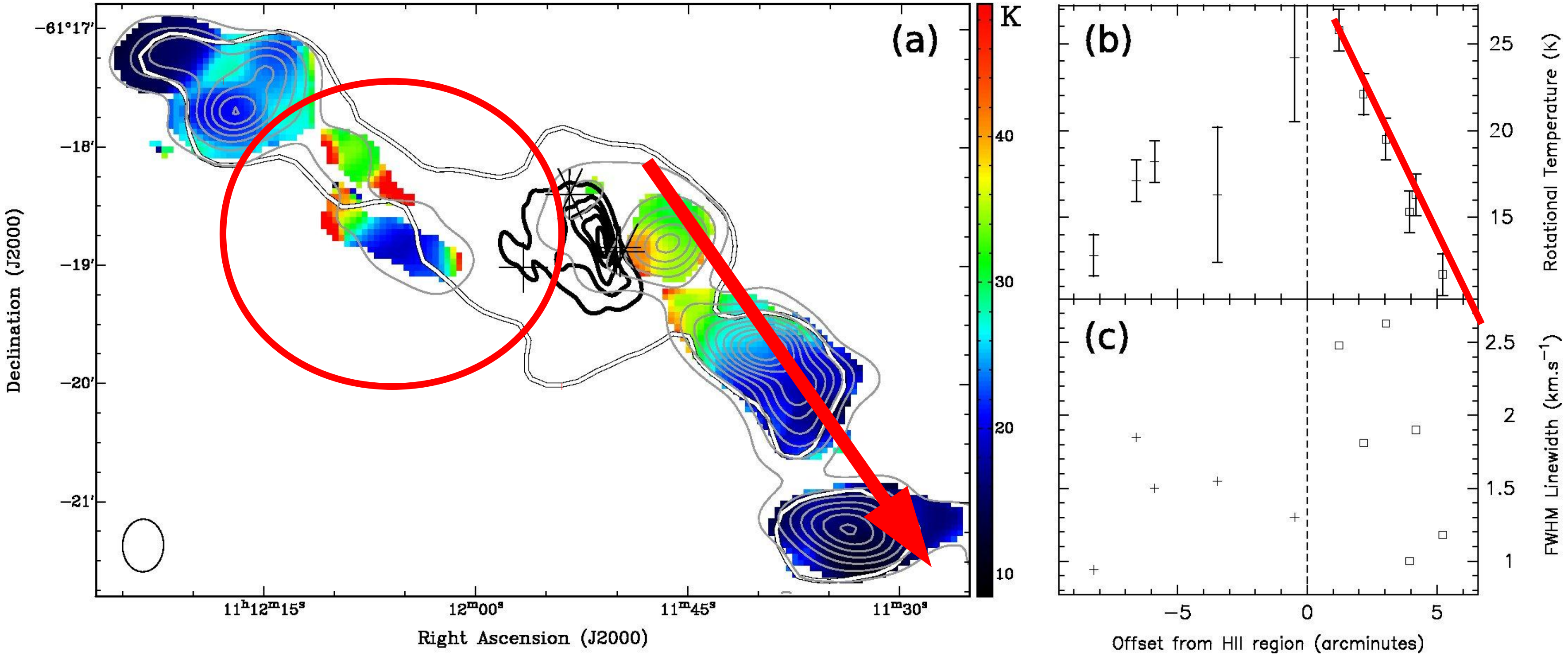

- Temperature gradient away from the HII region
- Hot spots in eastern arm + free-free emission
- Gas is being dispersed in east & heated in west

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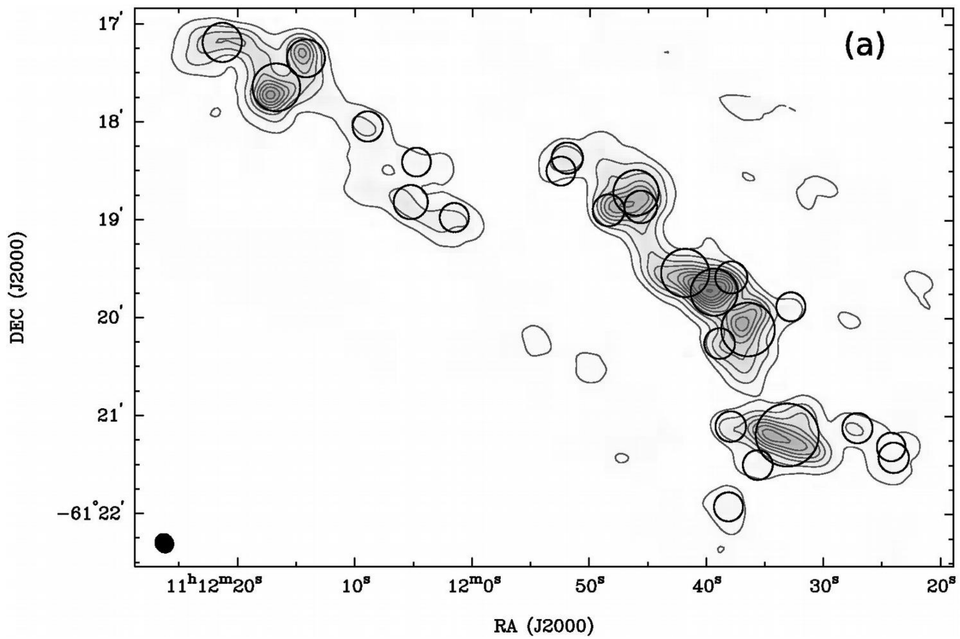

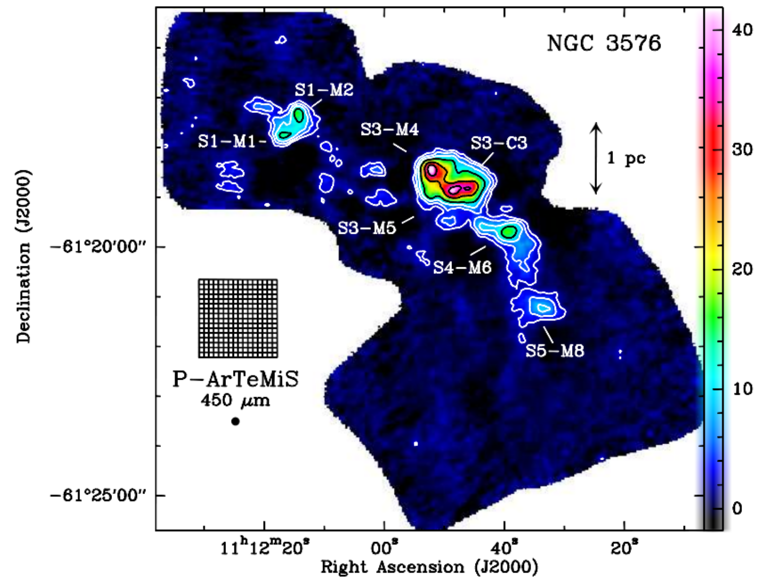

- FELLWALKER used to decompose emission into 'cores'
- 25 cores found, M = 5 → 500 solar masses
  - Values corrected for abundance variations by comparison to new 450 micron data from APEX (Andre 2008)

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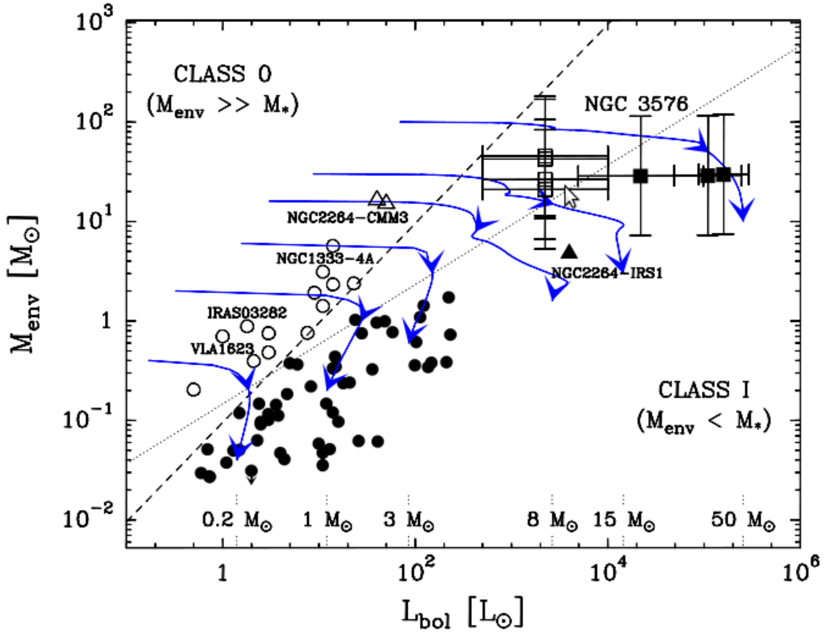
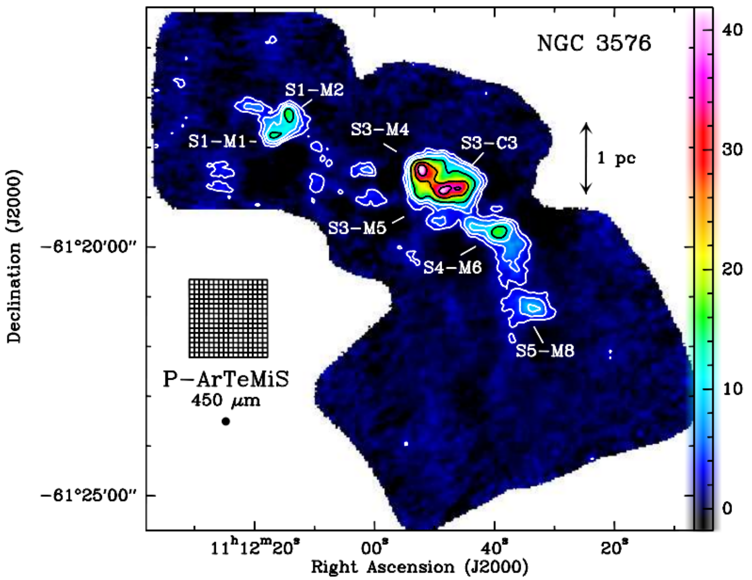
Dissecting a star-forming region





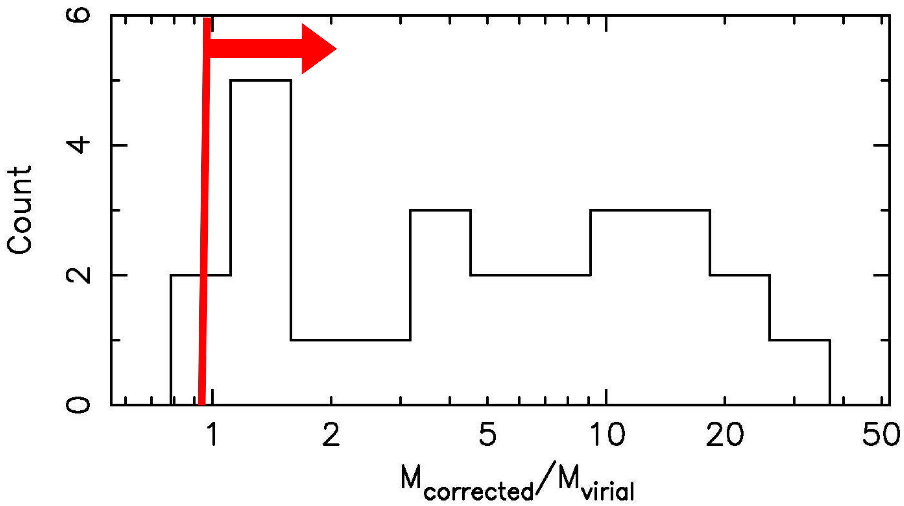
- FELLWALKER used to decompose emission into 'cores'
- 25 cores found, M = 5 → 500 solar masses
  - Values corrected for abundance variations by comparison to new 450 micron data from APEX (Andre 2008)
- Clump mass & luminosity suggest 8 – 50 M_{sun} stars are forming in each clump
  - Weak evidence for an evolutionary gradient

Case Study

Dissecting a star-forming region

- Virial masses:

$$M_{vir} = k\, r\, \Delta V^2$$



- Find that all cores are at least gravitationally bound
- Magnetic support:

$$B^2 - B_0^2 = \frac{9}{10}\left(1 - \frac{10}{9\,f}\right)\frac{G\,M^2\,\mu_0}{R^4\,\pi}$$

- Find that fields of 1 → 40 mG required
  - Higher than ~ 1 → 6 mG typically measured in MSF
  - Cores likely to be collapsing

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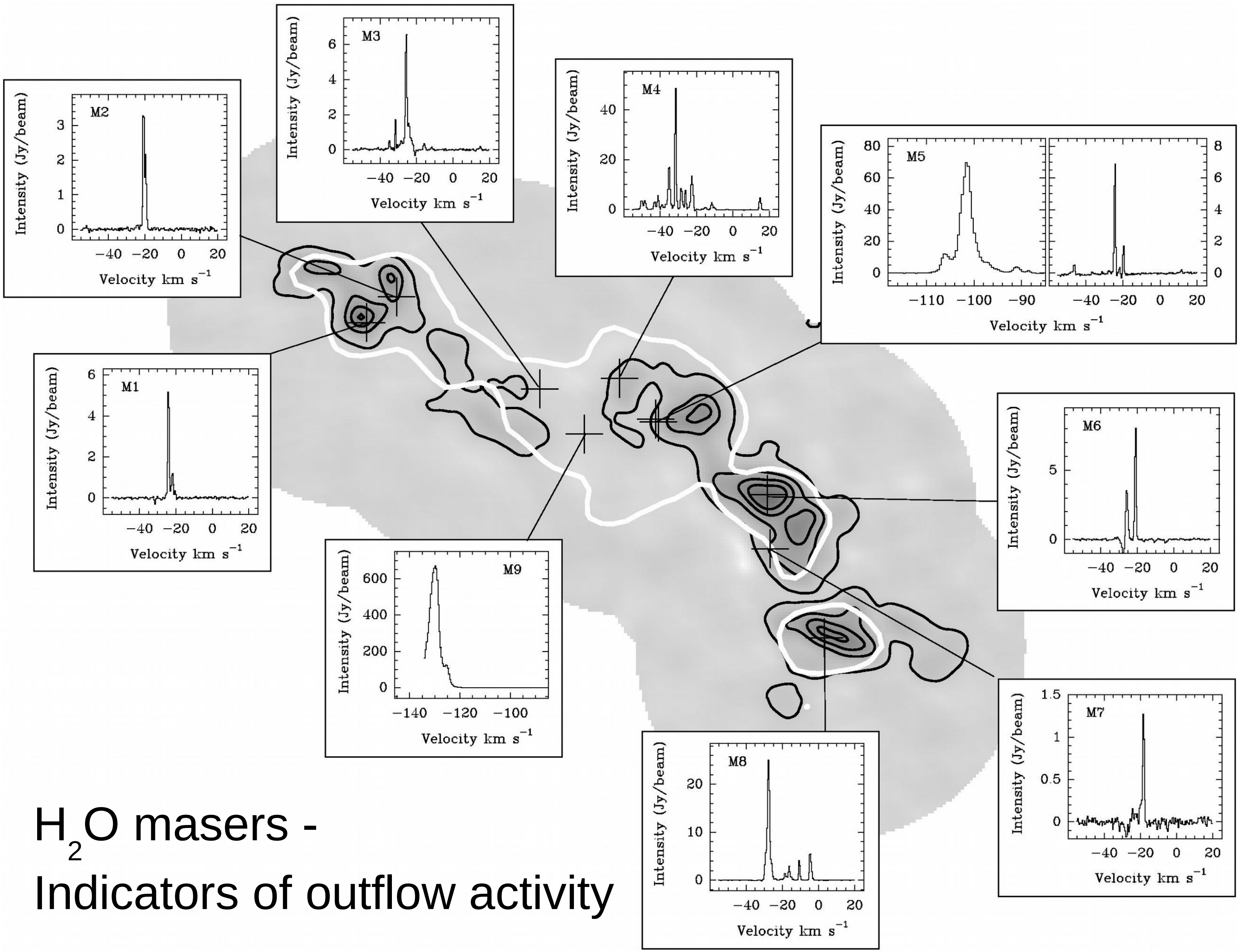

H_2O masers -

Indicators of outflow activity

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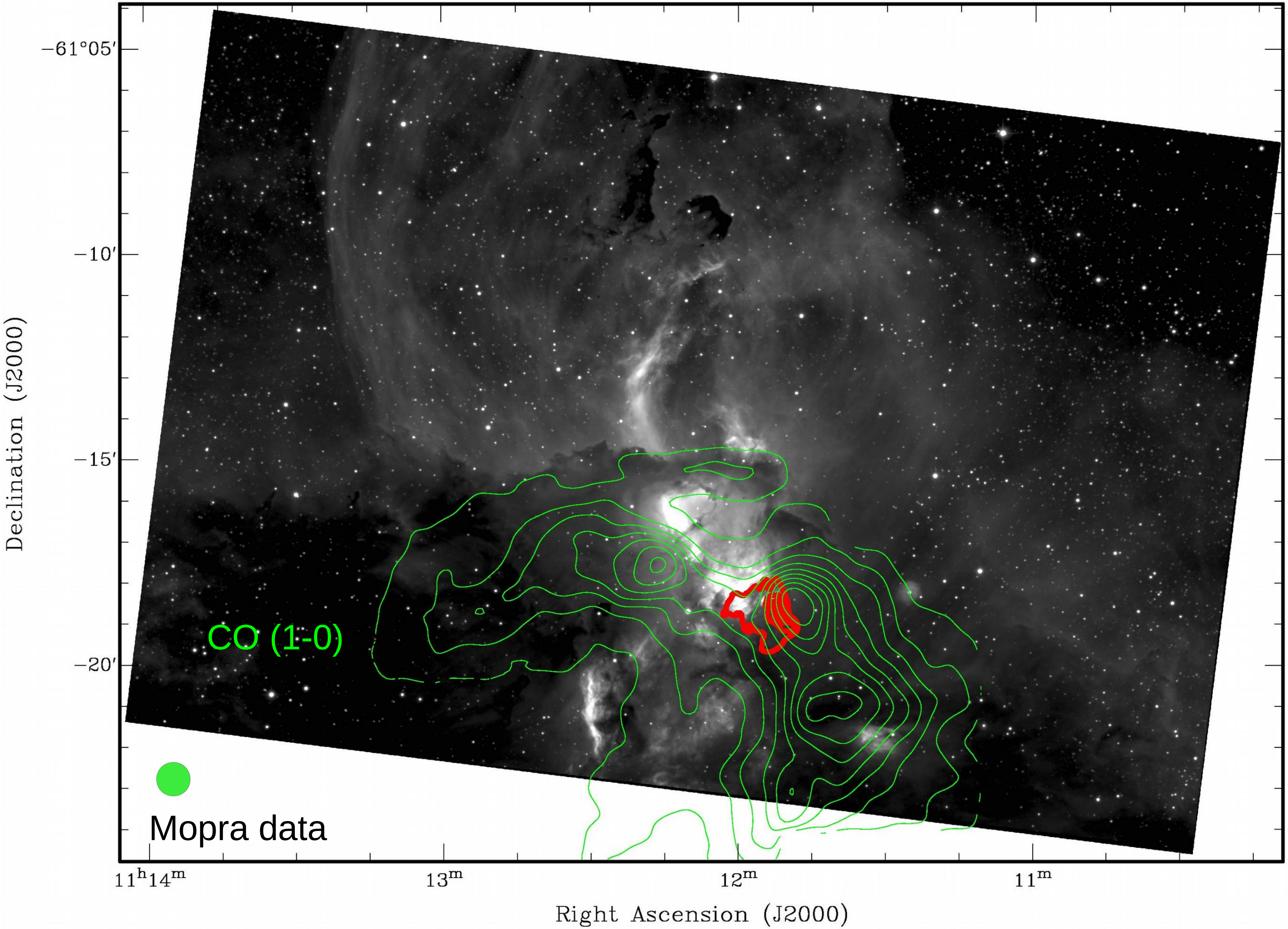
Dissecting a star-forming region



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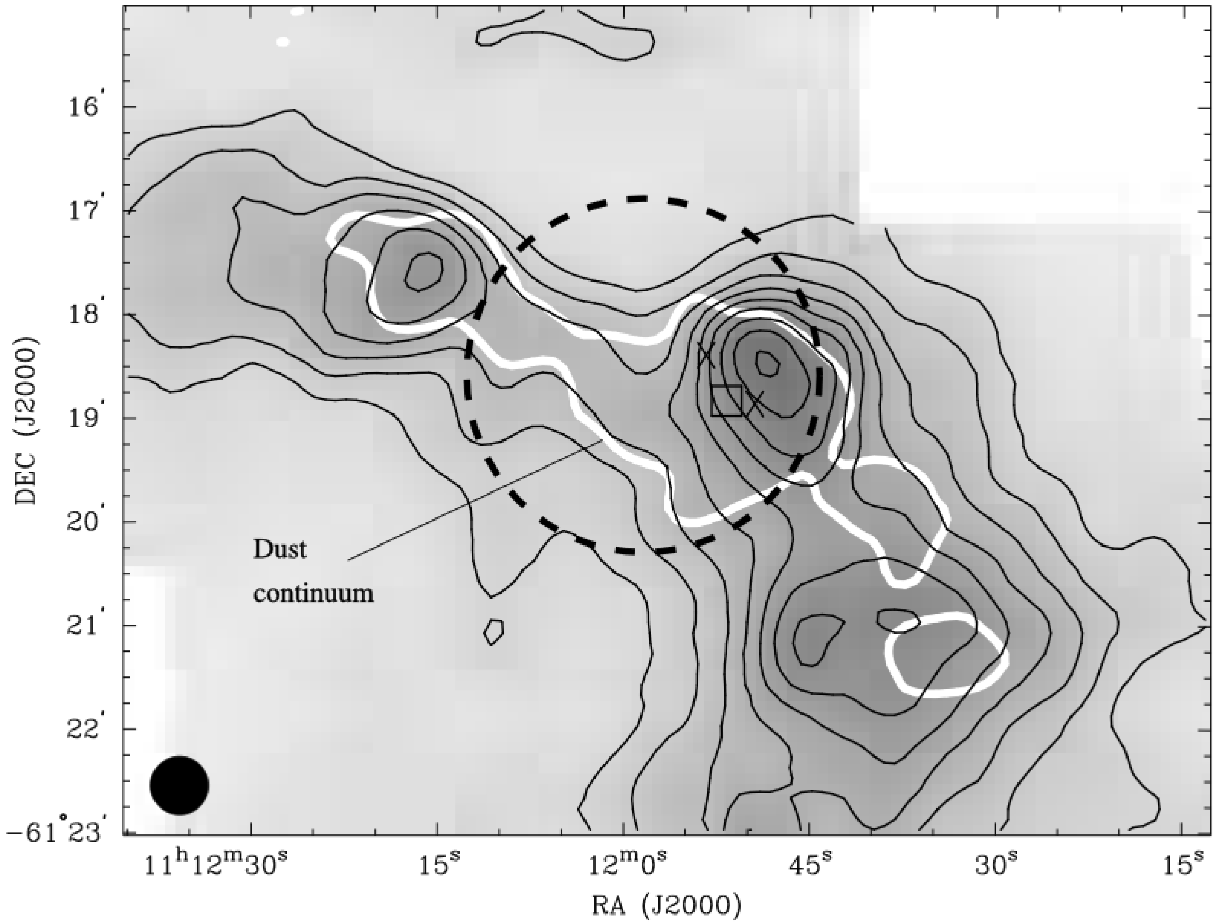

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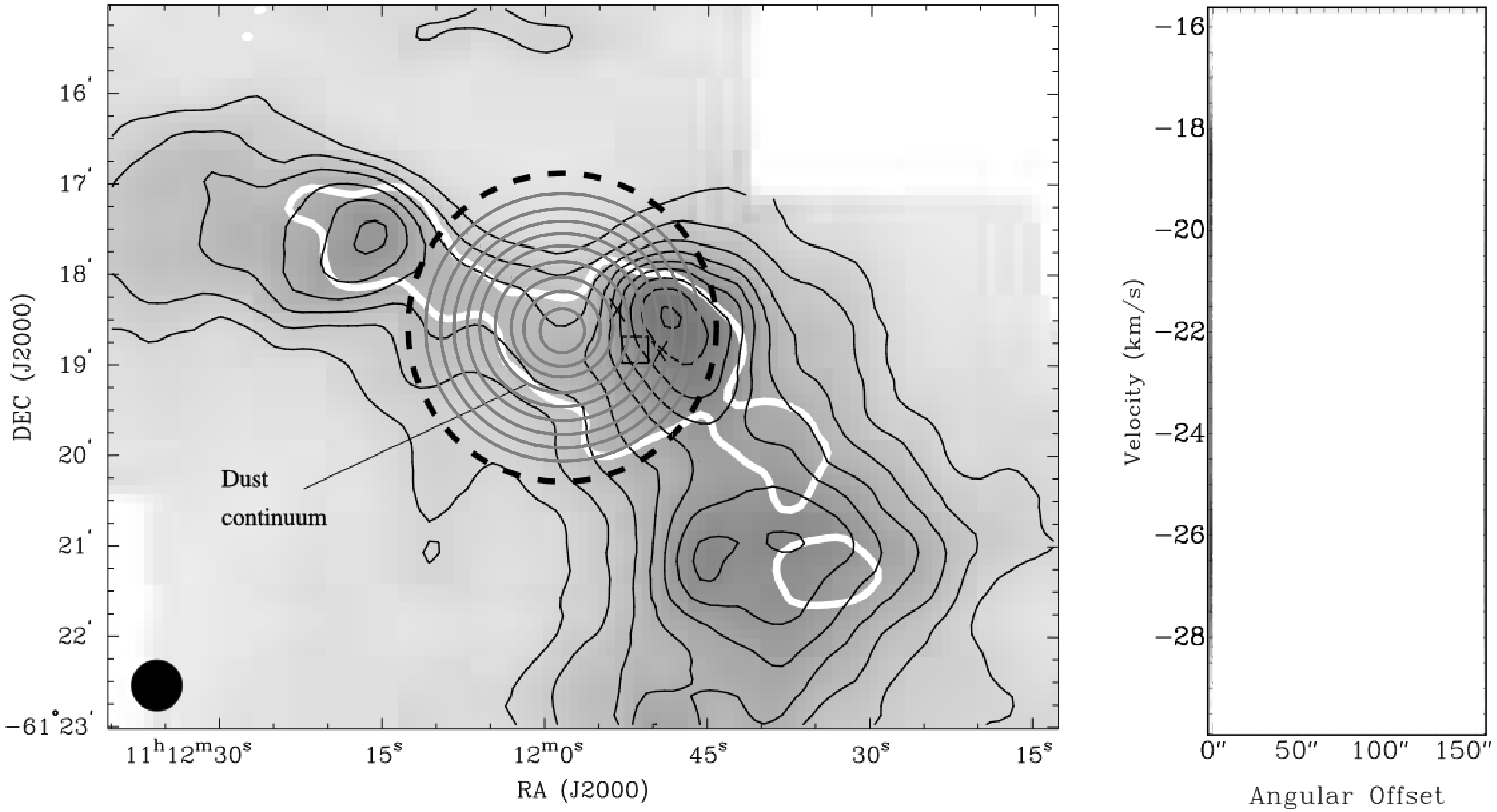

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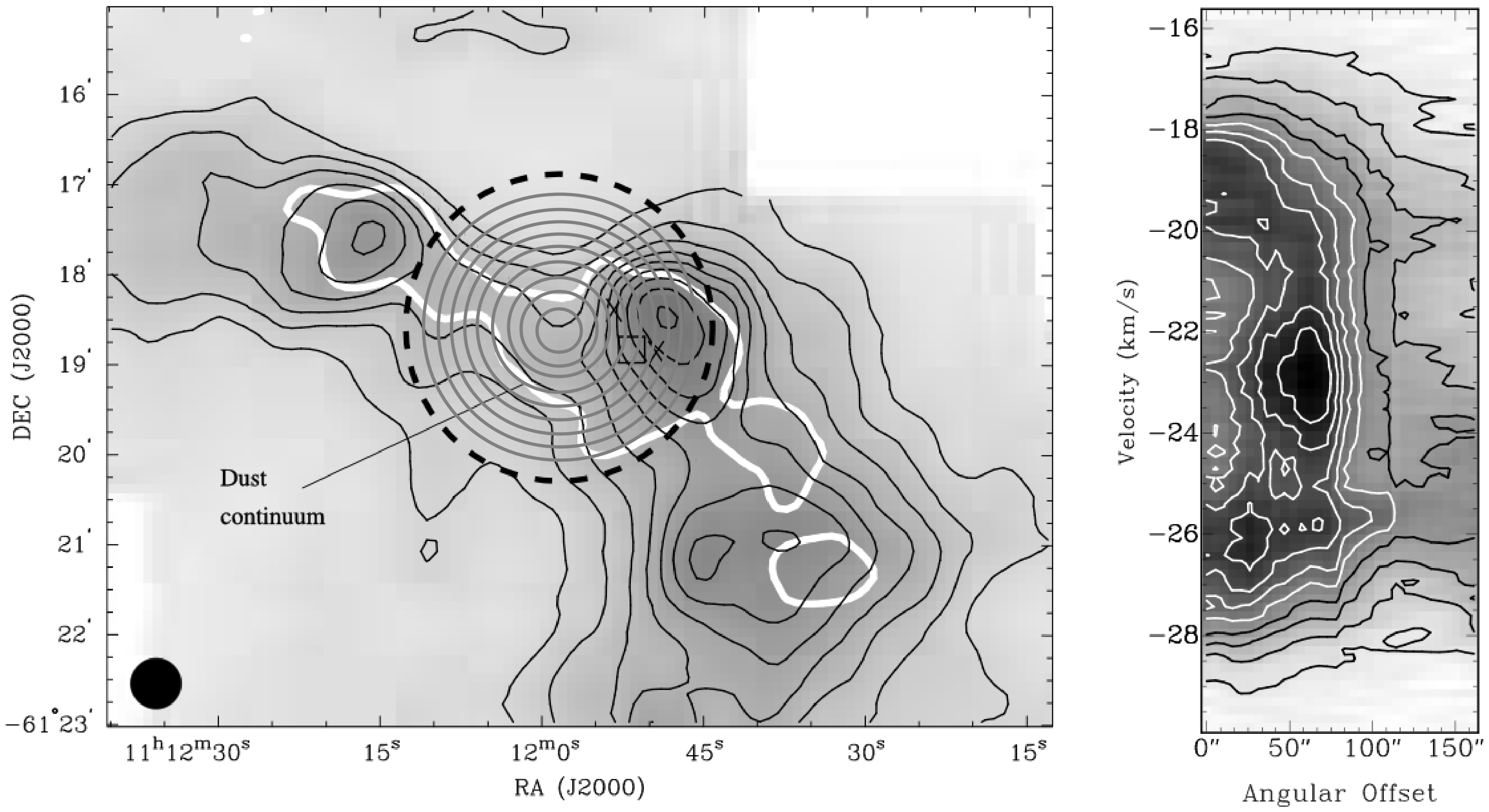

- Classic velocity signature of an expanding shell

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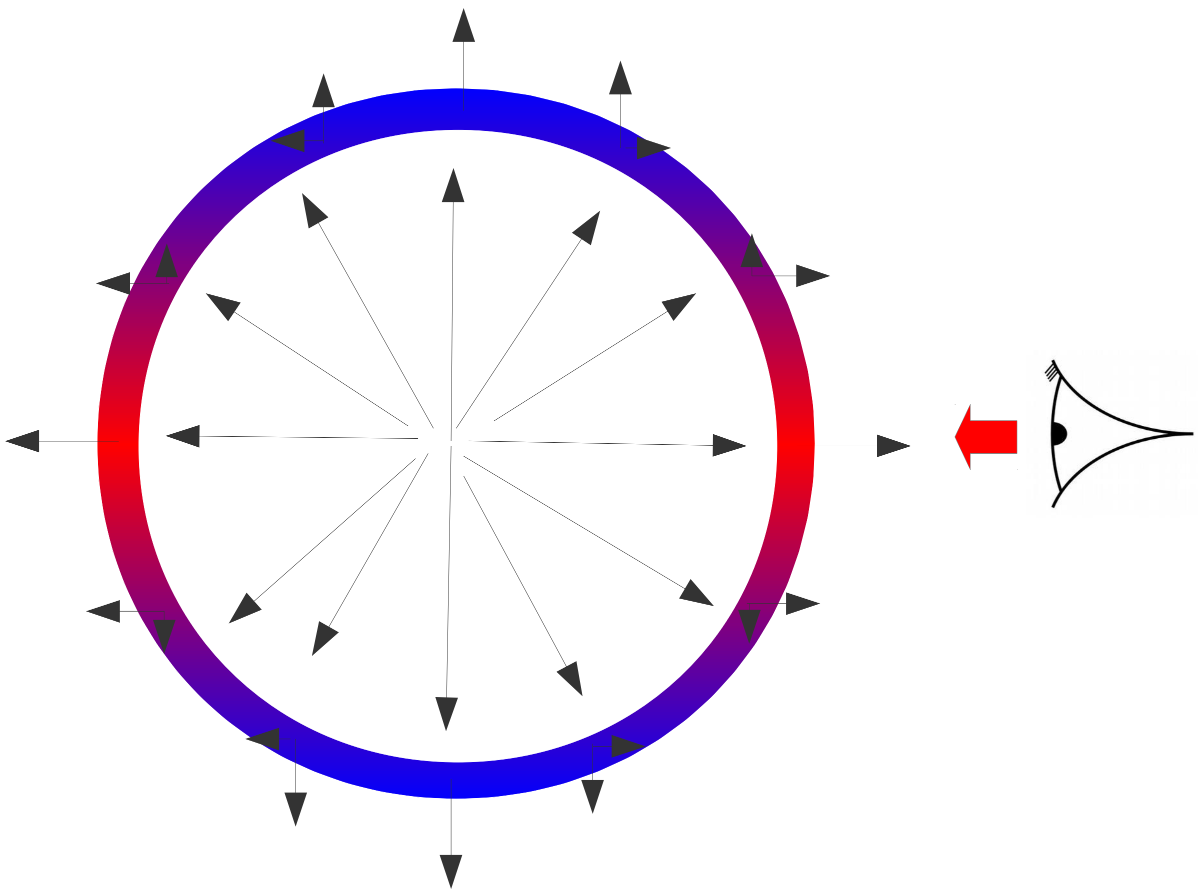

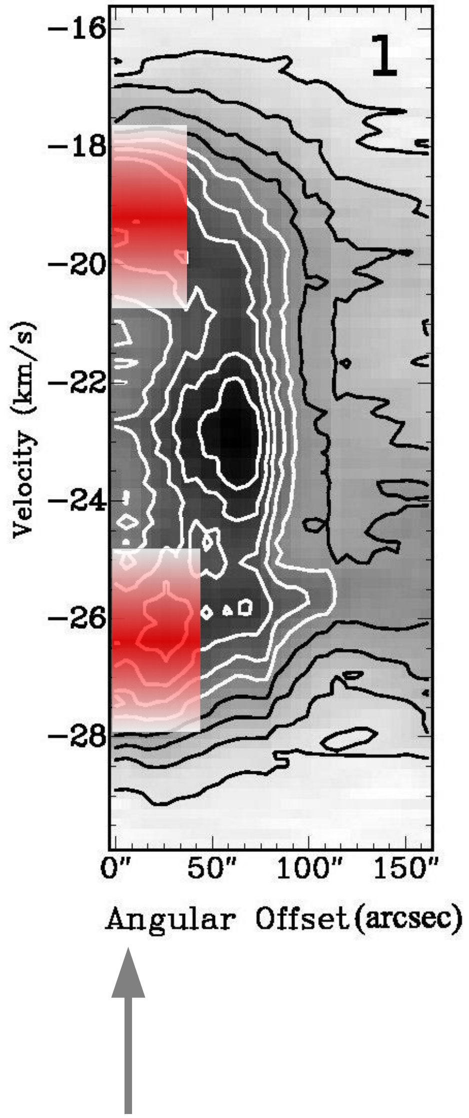

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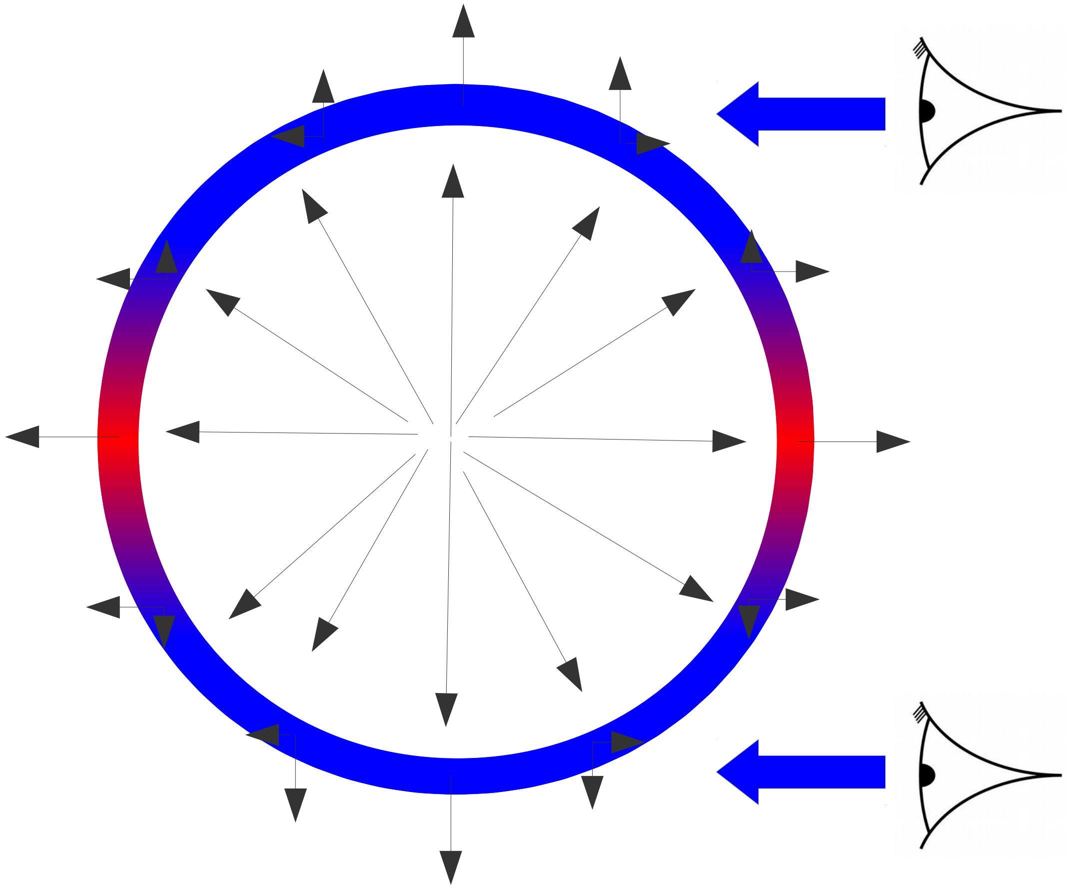

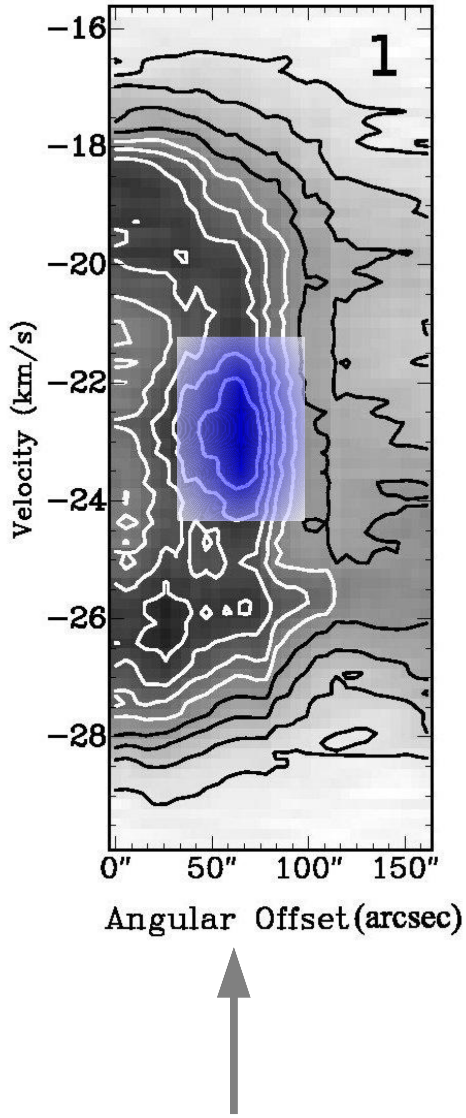

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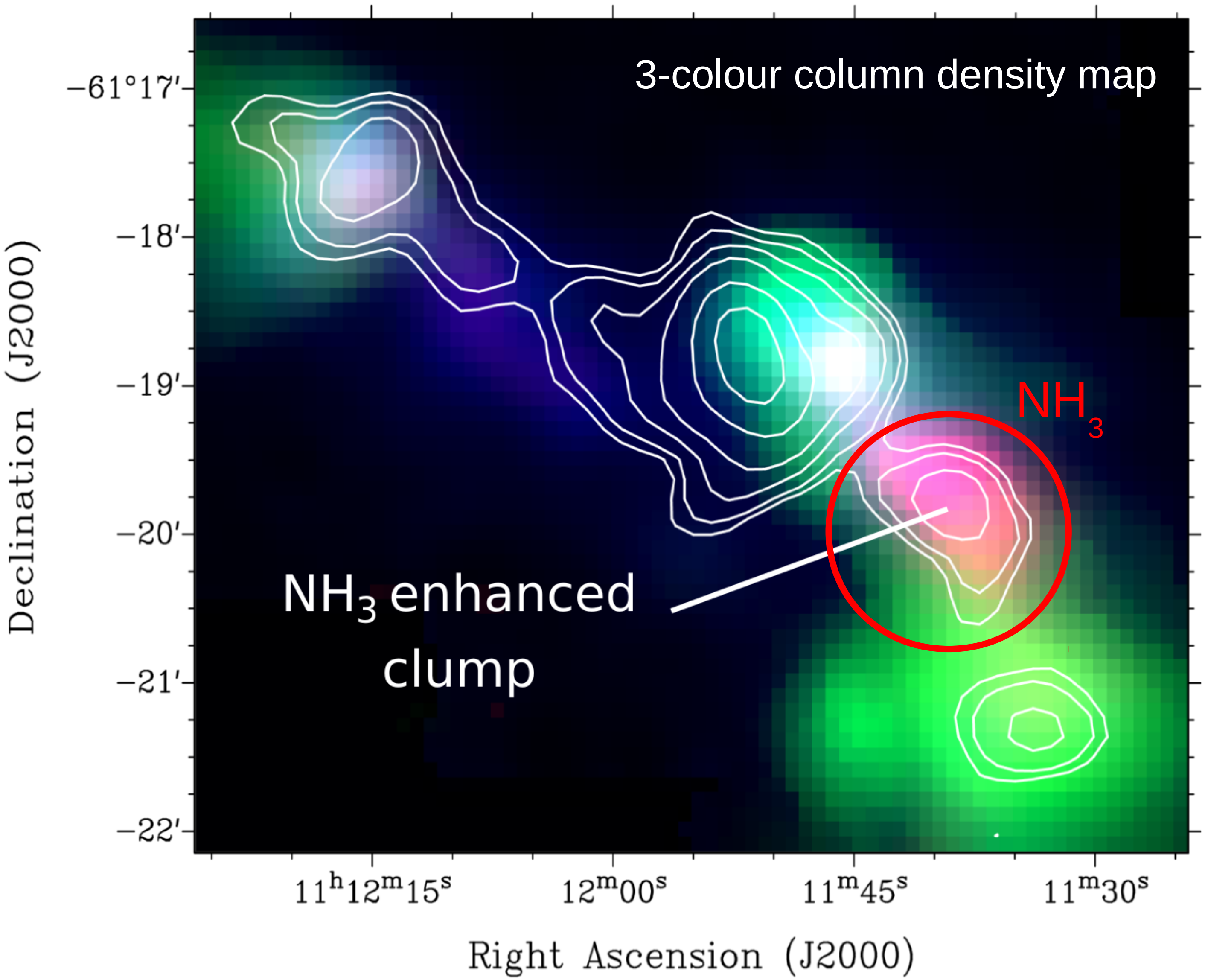

- Green = CO   Blue = CS   Red = N_2H^+

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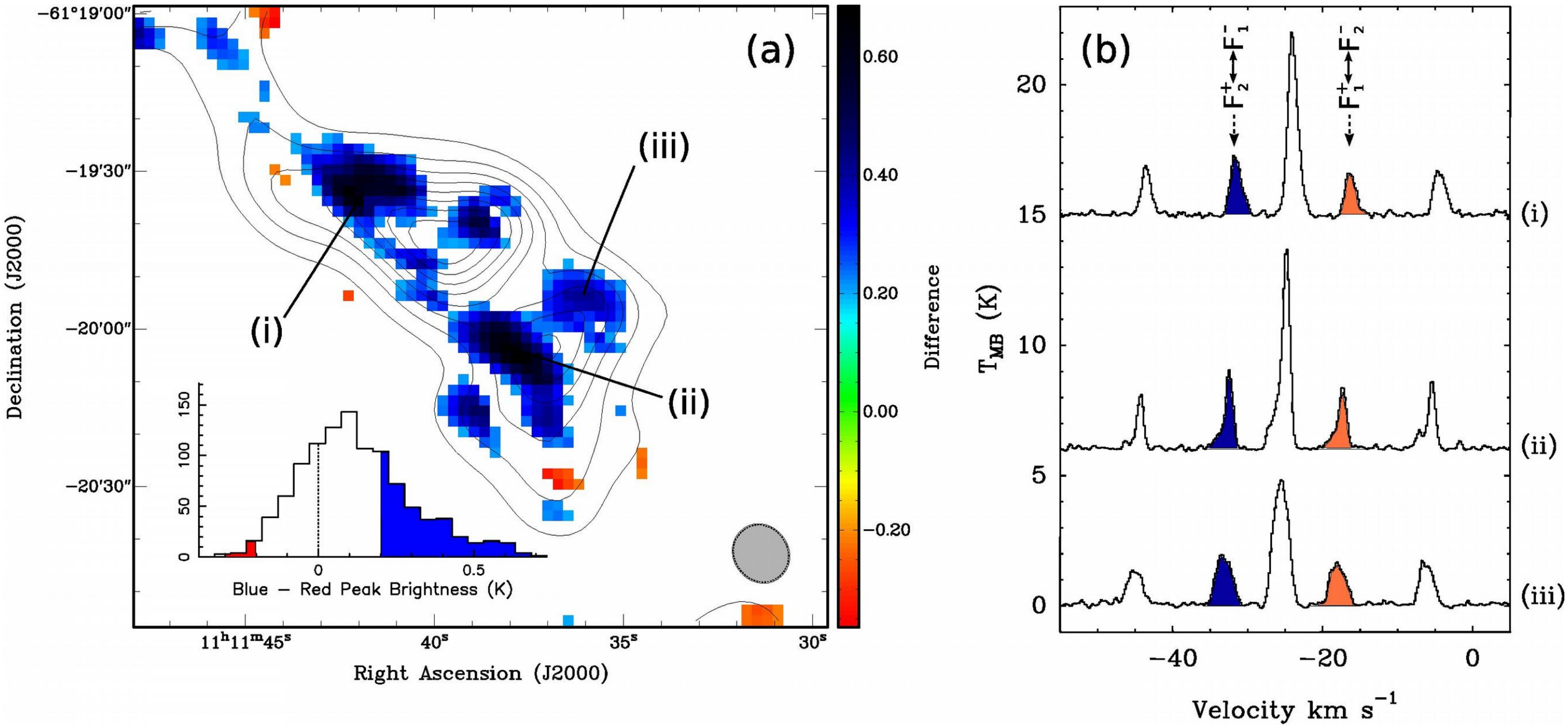

- Evidence for bulk gas motions seen in line profiles (Park '96)
- Clump is dominated by significantly blue-skewed profiles
  - Infalling gas motions

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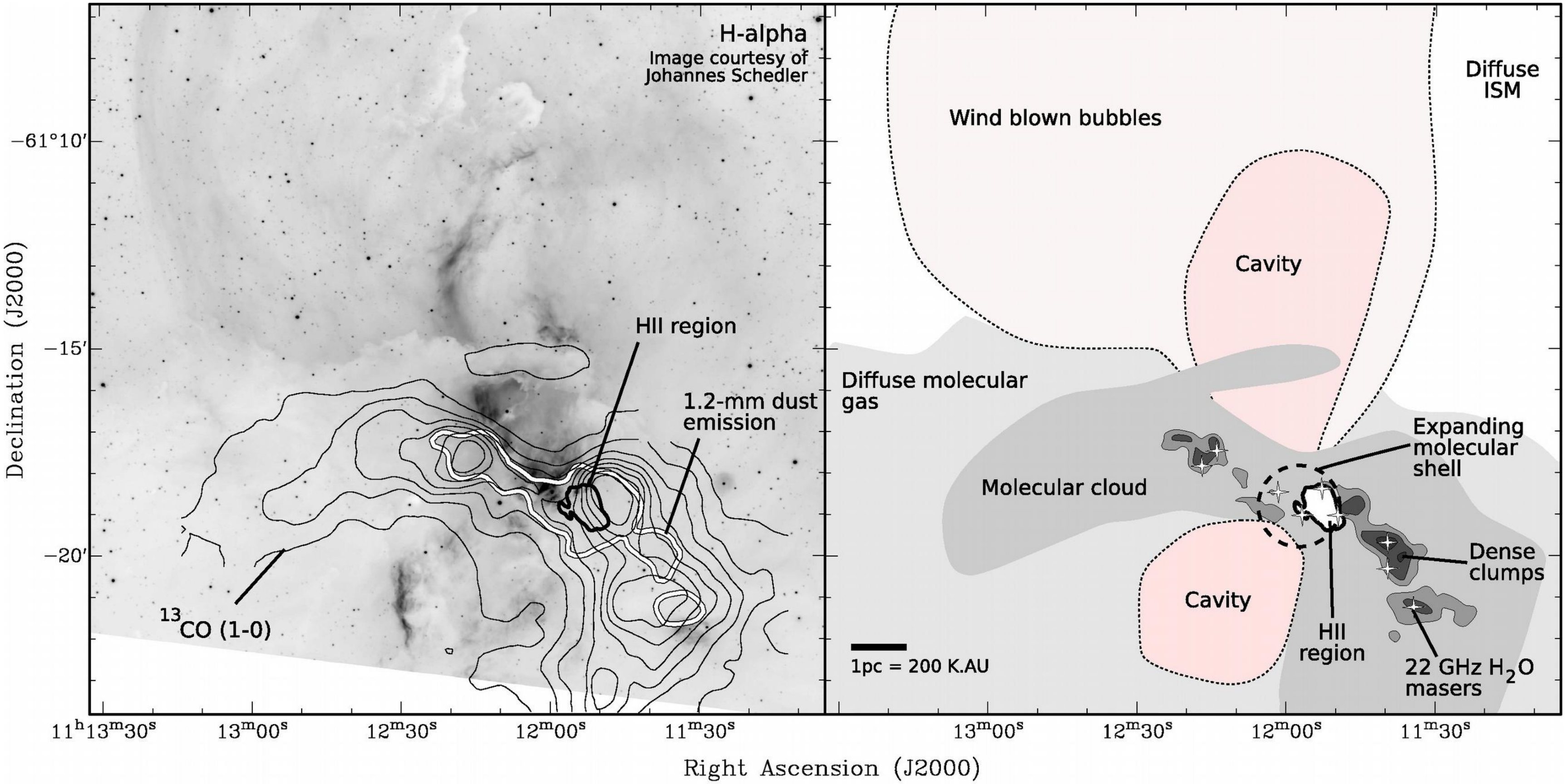
Dissecting a star-forming region



- HII region expanding into a dense molecular cloud
- Heating and/or dispersing the immediate environment
- Very young massive star formation observed in dark clumps
- Cores collapsing while also showing evidence of outflows
- Some evidence of an age gradient - triggering?

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Summary

- Molecular lines are an incredibly useful diagnostic of processes in the Galaxy
- With a few simple assumptions you can determine the physical and chemical and kinematic conditions in star-formation regions
- For a more detailed description please see the notes.

Thanks for listening!